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The Algebraic Galaxy of *Simple* Macroeconomic Models:
A Hitchhiker's Guide

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Abstract

Simple macroeconomic frameworks like the IS/LM and have survived because they help us conceptualize complex problems while also providing 'back of the envelope' estimates of macroeconomic outcomes. Herein, a bare-bones New Keynesian extension of the IS/LM model yields solutions for core macro variables (output gap, inflation, interest rate, real exchange rate misvaluation) -- expressed in *percent*. We then extend that standard model to also generate a corresponding set of demand-side elements -- expressed in *currency units*. A key aim of the paper is to reconcile these two metrics in ways that also aid communication and intuition – including through IS/LM-style graphs.

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"…if you stick a Babel fish in your ear you can instantly understand anything in any form of language."

Douglas Adams (1979).

Introduction

Paul Krugman (2000) argued that traditional, static macroeconomic models -- the IS/LM or one of its heirs -- perform some tasks better than more recent models that are based on dynamic intertemporal optimization precisely *because* they are so simple. More recently, Olivier Blanchard (2009) made a similar point when proposed the "re-legalization of shortcuts and of simple models..." He notes that "Approximating complex relations by simple ones helps intuition and communication."

Precisely what functions do such models fulfill? As an entry point for novice economists, including undergraduate students, the IS/LM remains the preferred teaching vehicle. For applied professionals, as Krugman noted, such a framework often serves as the default 'mental model' to guide policy discussions.

But, simple static models can do even more. A bare-bones version of the New Keynesian model – direct heir to the IS/LM – can generate equilibrium values of standard 'core' macroeconomic variables: the output gap, the interest rate, the inflation rate and the real exchange rate (percent deviation from norm). Such an extended 'back-of-the-envelope' calculation can serve as a cross-check against results from more complex methods.

By extension, we can also obtain the corresponding *expenditure components* of output -- consumption, investment, government, and net exports – expressed in *currency units* – Dollars, Euros, Pesos, and so on. Doing so helps us answer questions like "To what extent is investment crowded out?" or "Is the net export deficit too high?" But, some algebraic rescaling is required to reconcile these currency unit flows with the core variables which are typically measured in *percent*.

This paper examines whether such a rescaling can be done in a way that aids our intuition – in support of Blanchard's goal. One idea discussed herein is to recast simple linear expenditure equations (i.e. a Keynesian consumption function) in terms of potential output and the real rate of interest relative to some neutral rate. Recasting familiar equations in this way should help illustrate short-run and long-run economic relationships.

This paper does not present any new model *per se*. Instead, it suggests a way that different parts of previously existing (and quite standard) models can correspond with one another. The paper's title draws upon a literary reference. In Douglas Adams' (1979) novel "The Hitchhiker's Guide to the Galaxy," creatures from different planets can communicate with one another if they implant in their ear an animal that translates different languages – a "Babel fish." Hence the analogy: we seek a way for different units in the macroeconomic galaxy to speak with one another – a macroeconomic "Babel fish."

This paper was written with two key audiences in mind. First, applied macroeconomists should find the framework useful in their day-to-day assessments. The methods are especially useful for comparing alternative scenarios for one or several periods. The open-economy model illustrated herein would be especially useful for a financial programming exercise – scenarios which fully illustrate prospective values for both expenditure and financing flows. This idea is not new: Khan (1987) argued for the inclusion of a model within the International Monetary Fund's financial programming exercise as a way to assess both economic risks and policies required to eliminate macroeconomic imbalances. [1]

Second, instructors of macroeconomics at the intermediate level should find the methods discussed herein transparent enough to convey to their students – perhaps as a supplement to their current text. For both audiences, the author has developed companion spreadsheets that are available on line.[2]

The paper is organized as follows. In Part II, we begin with the case of the closed economy, developing and re-interpreting the standard linear expenditure equations: consumption, investment, and government spending. In Part III, we re-derive the core macroeconomic variables and their equilibrium counterparts on the expenditure side. In Part IV, we extend the model to the open economy. In Part V, we discuss how the long-run expenditures shares in such a model might be determined by appealing to microeconomic reasoning. In Part VI, e extend the 'Babel fish' analogy to match up the traditional currency unit equations with those expressed in natural logarithms or percentage changes; such a procedure was used in a module to help teach financial programming and policies (FPP) that was developed by the

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[1] Ordinarily, the financial programming exercise is conducted over a one-year horizon. For this reason, the model developed herein has no lags – but might be extended to include them.

[2] Other papers that propose extensions of the traditional IS/LM framework for the undergraduate classroom include, Romer (2000), Walsh (2002), Weerapana (2003), and Carlin and Soskice (2005).

International Monetary Fund's Institute For Capacity Development (IMF/ICD) (see also the Appendix). Part VII concludes.

II Expenditures in the Closed Economy

We begin with several linear equations that describe expenditures in a closed economy. Consumption C_t is described by a standard (Keynesian) consumption function:

$$C_t = a_{C0} + a_{CY}(Y_t - T_t) \tag{1a}$$

where Y_t denotes output, T_t denotes tax revenue taken by the government and a_{CY} is the marginal propensity to consume out of disposable income $Y_t - T_t$, $0 < a_{CY} < 1$.
We assume that tax revenue itself is a constant fraction of output τ plus a temporary lump-sum component "tax policy" component, TP_t:

$$T_t = \tau Y_t + TP_t \tag{1b}$$

Thus the consumption function is now:

$$C_t = a_{C0} + a_{CY}(Y_t(1-\tau) - TP_t) \tag{1c}$$

Next, the level of investment I_t is a function the real interest rate r_t:

$$I_t = a_{I0} + a_{Ir} r_t \tag{2}$$

where $a_{Ir} < 0$.

Finally, government expenditure is some normal level (a constant), namely a_{G0} plus a "government policy" shock, GP_t:

$$G_t = a_{G0} + GP_t \tag{3}$$

In this example we assume government spending to be essentially useless. However, that assumption is not essential and may be modified in an extension of the model.

In the closed economy, output equals the sum of these three components:

$$Y_t\big|_{Closed} = C_t + I_t + G_t \tag{4}$$

II.a Consumption: A more revealing view

Equations (1a-c) summarize the tried-and-true consumption function that has appeared in many textbooks over the years. Unfortunately, such a formulation fails to bring out some of the important ideas that economists have developed to understand the household's consumption decision. For example, a life-cycle (LC) approach stresses that a rational household consumption will use some estimate of its long –run disposable income – not just its current value – when it decides how much to save and consume.

In a similar vein, a strict interpretation of the permanent income hypothesis (PIH, Friedman, 1956) suggests that households will refrain from changing their level of consumption when their income deviates from its permanent value on a transitory basis. However, there may be 'hand-to-mouth' or 'liquidity constrained' households whose consumption expenditures do vary with transitory income.

Fortunately, we can adapt that traditional consumption function to incorporate these ideas. As a first step, we assume some level of potential output in any period, Y^P. Second, to obtain proxy measures of permanent and transitory disposable income, we maintain the constant tax rate assumption and we add and subtract τY^P from (1b) to obtain:

$$T_t = \tau Y^P + \tau(Y_t - Y^P) + TP_t \tag{5}$$

Thus, the cyclical component of taxes would be the second term on the right hand side, $\tau(Y_t - Y^P)$. [3] We may now add and subtract the term $a_{CY}(1-\tau)Y^P$ from the right-hand side of consumption function (1c) and rearrange to obtain:

$$C_t = \tilde{a}_{C0} + a_{CY}\{[(1-\tau)(Y_t - Y^P)] - TP_t\} \tag{6a}$$

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[3] This simple version of the model does not include automatic stabilizers. To do so is easy: we just assume a cyclical tax rate τ_{cyc} that is negative. In this case, the second term in equation (5) becomes $\tau_{cyc}(Y_t - Y^P)$ whose positive value when output is below potential tells us the size of government safety-net transfers.

where $\tilde{a}_{C0} = a_{C0} + a_{CY}(1-\tau)Y^P$. It will be useful to redefine this constant term in the equation as: $\tilde{a}_{C0} = Y^P(1-\sigma)*(1-\tau)$ where is the economy's long-run rate if saving rate. Thus, linear equation (1) is now reinterpreted as:

$$C_t = \underbrace{(1-\tilde{\sigma})Y^P}_{\text{long-run component}} + (1-\sigma_{cyc})\underbrace{\{[(1-\tau)(Y_t - Y^P)] - TP_t\}}_{\text{short-run component}} \tag{6b}$$

where $(1-\tilde{\sigma}) = (1-\sigma)*(1-\tau)$. The interpretational advantages of (6b) over (1c) should be readily apparent. In both equations, the first term on the right-hand side is a constant. In the rescaled equation (6b), that constant *explicitly* informs us about household consumption and disposable income in the long run. The parameter $\tilde{\sigma}$ may be thought of as the long-run savings rate (adjusted for taxes). Such a parameter can be clearly traced back to a long-run growth model that includes taxes -- either the Solow model or (as discussed below in Part VI) one based on intertemporal optimization.

The second term on the right-hand side tells us the short-run or cyclical component of consumption –including the effect of one-off tax measures. According to a strict interpretation of the permanent income hypothesis, consumers save the entirety of any temporary windfall: under this interpretation, σ_{cyc} would be unity. However, under a less stringent interpretation, there are some hand-to-mouth (or liquidity constrained consumers). Therefore, under a less strict reading of the PIH, $0 < \sigma_{cyc} < 1$. Put differently $(1-\sigma_{cyc})$ is simply an alternative name for the marginal propensity to consume out of transitory income.

Likewise, equation (6b) can also accommodate a continuum of views on the Ricardian Equivalence Hypothesis (REH). Since TP_t is defined as a one-off tax change, all else equal, a strict reading of the REH would also imply $(1-\sigma_{cyc})$ =0.

II.b Rescaling Investment and Government Spending

Consider next a reinterpretation of the investment function (2). We assume that there is a natural (or neutral) real rate of interest $\bar{r}$ which, in the absence of any other shocks to the economy, yields a zero output gap. We may interpret $\bar{r}$ as an interest rate which would hold in a steady state: the marginal product of capital at a steady state minus the depreciation rate. *Also, as we confirm in Part V.a below,* in a model based on intertemporal optimization by a representative consumer, this steady-state interest rate approaches the subjective rate of time preference. We may subtract and add $a_{Ir}\bar{r}$ to the right-hand side of that equation and rearrange to obtain:

(7)

$$I_t = \tilde{a}_{I0} + a_{Ir}(r_t - \overline{r})$$

where $\tilde{a}_{I0} = a_{I0} + a_{Ir}\overline{r}$. Again, it will be convenient to reinterpret $\tilde{a}_{I0}$ in terms of potential output. In a closed economy, investment exactly equals saving, therefore then $\tilde{a}_{I0}$ must equal $\psi Y^P = \sigma * (1-\tau)Y^P$. Likewise, the national income identity requires that the constant in the government expenditure equation be: $a_{G0} = \tau Y^P$.

II.c Deriving the IS Curve – a rescaled approach.

In most textbooks, the IS curve is derived in levels by substituting equations (1c), (2) and (3) into identity (4) and solving for the equilibrium level of output (in currency units) as a function of the interest rate (negative) and fiscal policy shocks. In the rescaled approach suggested here, we may instead obtain the IS curve whose where output and the interest rate are measured as percentage gaps from potential output and the neutral interest rate, and the fiscal shocks are measured as a percent of potential.

That is, by substituting (6b), (7) and (3) into (4) and the rearranging, we obtain:

$$Y_t = Y^P * [1 + (1-\sigma_{cyc})\{[(1-\tau)(gap_t)] - tp_t\} + \varphi_{Ir}(r_t - \overline{r}) + gp_t] \qquad (8)$$

where $gap_t = Y_t / Y^P - 1$, $\varphi_{Ir} = a_{Ir} / Y^P$ is a response parameter that is scaled to potential output and both fiscal shocks are also expressed as a percent of potential: $tp_t = TP_t / Y^P$ and $gp_t = GP_t / Y^P$.

We then subtract and divide both sides of the equation and fully solve to obtain an expression for the output gap IS curve:

$$gap_t \Big|_{IS,\text{Closed}} = \frac{\varphi_{Ir}(r_t - \overline{r}) + gp_t - (1-\sigma_{cyc})tp_t}{\tilde{\sigma}_{cyc}} \qquad (9)$$

where $\tilde{\sigma}_{cyc} = 1 - (1-\sigma_{cyc})(1-\tau)$; that is $1/\tilde{\sigma}_{cyc}$ is the familiar Keynesian multiplier for a closed economy. Note that $\varphi_{Ir} / \tilde{\sigma}_{cyc} < 0$. This ensures that the IS curve will have its familiar negative slope. The equation also confirms that the IS curve will shift to the right when there is a *fiscal expansion*, that is when $gp_t - (1-\sigma_{cyc})tp_t > 0$.

III Equilibrium in a Closed Economy: Core and Expenditure Components

A New Keynesian model for a closed economy consists of three core equations: the IS curve, a monetary policy reaction function, and an aggregate supply or Phillips curve relationship. These three variables yield three equilibrium core values – the output gap, the interest rate (real and nominal) and the inflation rate.

The monetary policy reaction function is typically phrased along the lines of John Taylor's (1993) interest rate rule for central banks, which may be written as:

$$i_t\big|_{Closed} = \bar{r} + \pi^e + \beta_\pi(\pi_t - \pi^*) + \beta_{gap} gap_t + r_t^{DISC} \qquad (10)$$

where i_t is the nominal (policy) interest rate, π^e is the market's expectation of inflation, π_t is the realized rate of inflation, π^* is the central bank's target rate of inflation, and r_t^{DISC} captures any discretionary deviation of the central bank from the previous elements of the equation. The interpretation of this equation is an appealing one: the central bank has a *dual mandate* – to stabilize both prices and output.

In a similar vein, the inflation rate is determined by the following simple Phillips-curve relationship:

$$\pi_t = \pi^e + \frac{1}{\eta}(gap_t - ss_t) \qquad (11)$$

where $\eta > 0$ is the elasticity of aggregate supply in the short run (assuming some nominal rigidity in prices) and ss_t is a supply shock – a perturbation of the aggregate supply curve, measured in percent of potential output. The term inside the parenthesis on the right hand side may be thought of as a marginal cost term. [4]

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[4] To see this interpretation, consider the following expression for the quantity of goods and services supplied in the short run: $Y_t^S = Y^P[1 + \eta(P_t - P_t^e) + ss_t]$, where Y_t^S = the quantity of output supplied at time t, P_t is the price level at time t, P_t^e is the expected price level at time t (as of t-1), $\eta \geq 0$ is the short-run elasticity of supply with respect to the price level, and ss_t is a supply shock (in percent of potential output). That is, $ss_t > 0$ may be thought of as a reduction in the marginal cost of production – a level shift.

This equation is essentially Lucas' (1973) familiar supply function. If we normalize the price level in the previous period P_{t-1} to unity, we may add and subtract one from the term inside brackets to obtain: $Y_t^S = Y^P[1 + \eta(\pi_t - \pi_t^e) + ss_t]$. Then, after dividing both sides through by

To keep the model as simple as possible, inflation expectations π^e are assumed to be determined exogenously. This assumption permits us to calculate the output gap in a way that uses elementary algebra. First, note that the IS curve (9) may be flipped over to show the real interest rate rather than the output gap on the left-hand side:

$$r_t\Big|_{IS,Closed} = \bar{r} + \frac{gap_t * \tilde{\sigma}_{cyc} - gp_t + (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} \quad (12)$$

We then obtain the equilibrium real interest rate from monetary policy and the supply side by combining equations (10) and (11). The first step is to substitute in Phillips curve equation (11) into Taylor rule curve equation (10) to obtain a reduced-form expression for the nominal interest rate. This substitution has an appealing interpretation: the Phillips Curve (11) poses a *constraint* for the central bank whose goal is to stabilize prices and output. Then, subtract off (11) from this term to obtain an expression for the real interest rate consistent with both the central bank reaction function and the Phillips curve – the "RR" schedule:

$$r_t\Big|_{RR,Closed} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t - b_{RRss}ss_t + r_t^{DISC}$$

where $b_{RR\pi} = \beta_\pi$, $b_{RRgap} = \dfrac{(\beta_\pi - 1)}{\eta} + \beta_{gap}$, and $b_{RRss} = \dfrac{(\beta_\pi - 1)}{\eta}$.

Note that IS and RR curves are both in output gap/interest rate space. The familiar graphical depiction is shown in Figure 2. The equilibrium output gap is now obtained by equating combining the IS and RR curves:

$$gap_t^{eq}\Big|_{Closed} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + \dfrac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\dfrac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]} \quad (14)$$

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potential output and subtracting one from both sides, we note that $gap_t = Y_t^S / Y^P - 1$. By inverting that expression, we obtain the Phillips Curve in the text.

We then obtain the two remaining core variables by substituting in gap_t^{eq} along with the other exogenous variables into equations (13) and (11) – the equilibrium real interest rate and inflation rate, r_t^{eq} and π_t^{eq} respectively.

The equilibrium nominal interest rate is computed as $i_t^{eq} = r_t^{eq} + \pi_t^{eq}$ -- consistent with other equations in the model. Coefficient values shown in the example may be said to be *coherent* in the sense that movements in r_t^{DISC}, the nominal interest rate, and the real interest rate are all in the same direction. [5] That is, an exogenous rise in r_t^{DISC} will be only partially offset by reductions in the interest rate – those that are induced by changes in the output gap and inflation. Likewise, to obtain the expenditure side components in real currency units (Dollars), results for gap_t^{eq} and r_t^{eq} are inserted into equations (6b), (7) and (3) alongside the assumptions regarding tax and expenditure policy, gp_t and tp_t respectively.

Examples of all calculations are presented in Figures 1 and 2. (The spreadsheet that generated the table and graphs is available online at ________________). The top section of figure 1 shows assumed shocks for three scenarios. Under the baseline, there are no shocks. Under alternative scenario (i) government spending expands by 1% of potential output but the monetary authority remains on its initial Taylor rule. Alternative scenario (ii) retains that 1% fiscal expansion but assumes in addition that the monetary authority accommodates by setting an interest rate that is 1% lower than the Taylor rule: r_t^{DISC} =-1%. The impacts of shocks on the output gap are shown in the lower 2/3rd of the figure. Under alt(i), we see that the government spending multiplier which takes into account the endogenous response of monetary policy is .7 (the fiscal component of output gap is 0.7% in response to the initial 1% shock). Under alt(ii), the fiscal component of the shock is equal to alt(i), but the effect of the monetary loosening – a movement *along* the IS curve – is an additional 0.4%. Thus, the output gap under is 0.7% under alt(i) but 1.1% under alt(ii).

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[5] The necessary and sufficient conditions for such coherence are: $[\frac{b_{RRgap}}{den} + 1] > 0$ for the real interest rate and $\{\frac{b_\pi}{\eta * den} + \frac{b_{gap}}{den} + 1\} > 0$ for the nominal interest rate, where $den = \left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]$ is the denominator term in equation (14).

Figure 1
Closed economy model
Shocks and output gap

In percent of potential output		Shocks -- Expenditure base	alt(i)	alt(ii)
Gov't Spending	gp_t	0.0%	1.0%	1.0%
Tax Measures (one-off)	tp_t	0.0%	0.0%	0.0%

In percent		Shocks - Supply / Expected inflation		
Supply shock (% of Y^P)	ss_t	0.0%	0.0%	0.0%
Inflation expectations (gap w.r.t. target)	$\pi^e - \pi^*$	0.0%	0.0%	0.0%

In percent		Shocks - Discretionary monetary policy		
Deviation from Taylor Rule (shift)	r_t^{DISC}	0.0%	0.0%	-1.0%

Calculation of equlibrium output gap -- component by component

(a) Inflation expectations component

$$\text{inflation expectations component} = \frac{b_{RR\pi}(\pi^e - \pi^*)}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

0.0%	0.0%	0.0%

(b) Supply shock component

$$\text{supply shock component} = \frac{b_{RRss} ss_t}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

0.0%	0.0%	0.0%

(c) Fiscal component

$$\text{fiscal component} = \frac{\frac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

0.0%	0.7%	0.7%

(d) Discretionary monetary component

$$\text{discretionary monetary policy} = \frac{r_t^{DISC}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

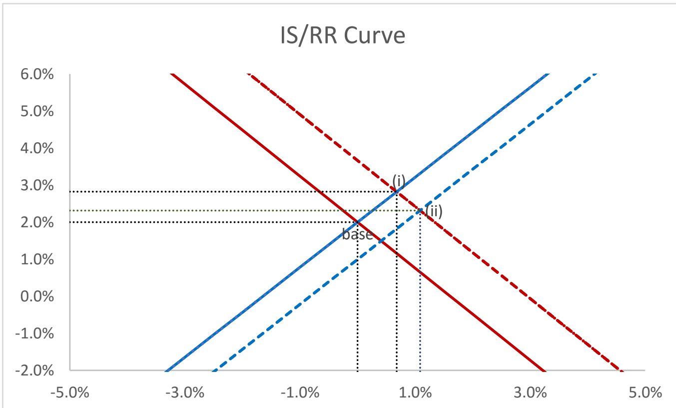
0.0%	0.0%	0.4%

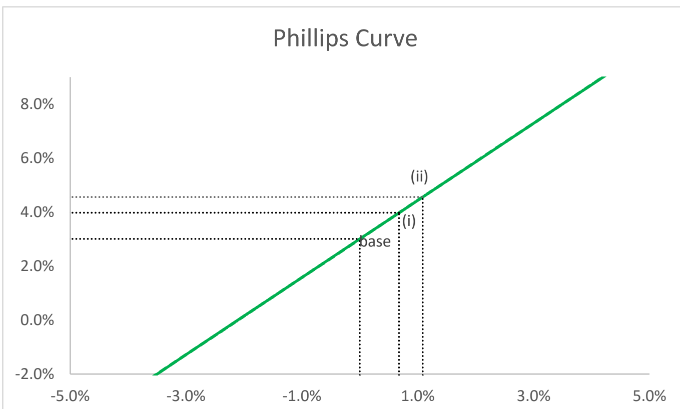
Output gap (a)-(b)+(c)+(d)

0.0%	0.7%	1.1%

$$gap_t^{eq}\big|_{Closed} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss} ss_t + \frac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

Figure 2
Closed Economy Model:
Graphics, core variables and expenditure components





alt(i) – fiscal expansion, central bank remains on Taylor Rule

alt(ii) – fiscal expansion, monetary accommodation discretionary loosening off Taylor Rule.

Output gap (a)-(b)+(c)+(d)

$$gap_t^{eq}\Big|_{Closed} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + \dfrac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\dfrac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$$

Equilibrium inflation rate

$$\pi_t = \pi^e + \frac{1}{\eta}(gap_t^{eq} - ss_t)$$

Equilibrium real interest rate

$$r_t^{eq}\Big|_{RR,Closed} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t^{eq} - b_{RRss}ss_t + r_t^{DISC}$$

Output gap (a)-(b)+(c)+(d)	0.0%	0.7%	1.1%
Equilibrium inflation rate	3.0%	4.0%	4.6%
Equilibrium real interest rate	2.0%	2.8%	2.3%
Currency-unit results			
Gross Domestic Product	16.79	16.90	16.97
Consumption	11.21	11.24	11.26
Investment	3.06	2.97	3.03
Governerment Spending	2.52	2.69	2.69
Demand side decomposition of output gap (percent of potential)			
Output gap	0.0%	0.7%	1.1%
Consumption	0.0%	0.2%	0.3%
Investment	0.0%	-0.5%	-0.2%
Governerment Spending	0.0%	1.0%	1.0%

Figure 2 shows the graphical representation of these scenarios. The upper diagram shows the IS and RR schedules in output gap/real interest rate space. Under alt (i) the downward sloping red line shifts to the right, from the solid to the dotted/dashed line, along the RR curve which remains in its original position. Thus, we see the joint increase of output and the real interest rate under this scenario. Under alt(ii), we see no further shift of the IS curve but the RR curve now shifts to the right. Accordingly, output under alt(ii) is higher but the interest rate is lower than under alt(i). Directly below, we see the Phillips Curve outcome: inflation rises under alt(i) and higher still under alt(ii) – as reflected in shifts along the green PC schedule.

Under the graphs, a table shows the results for both the core variables and the expenditure components. The bottommost table shows the contribution to the output gap by expenditure component. For example, under alt(i), we see that when government spending rises by 1%, there is an induced increase of consumption of 0.2% but higher interest rates help crowd-out investment by 0.5%. Under alt(ii), since interest rates are lower, the investment crowding-out component drops to 0.2% while consumption rises even further – an additional induced increase of 0.1% above alt(i).

IV The Small Open Economy: Exports, Imports, and External Financial Pressures

The framework may be extended to show the impacts that internal and external shocks will have on a small open economy – including core variables, the real exchange rate, and net exports. The first step is to expand the goods market to include both exports and imports. Thus, the output identity in an open economy rewritten:

$$Y_t\Big|_{Open} = C_t + I_t + G_t + X_t - IM_t \tag{15}$$

where X_t and IM_t denote exports and imports of goods and services, respectively.

However, there will also be trade in assets. Here we assume the economy to be small relative to the rest of the world. It faces an external rate of interest (including a risk premium) that it has no influence over. Importantly, the model can help us understand in a straightforward way how externally-based shocks, in addition to domestic shocks, will impact the economy's short-run equilibrium. As illustrated in figure 3, such effects will be transmitted through the export and import markets.

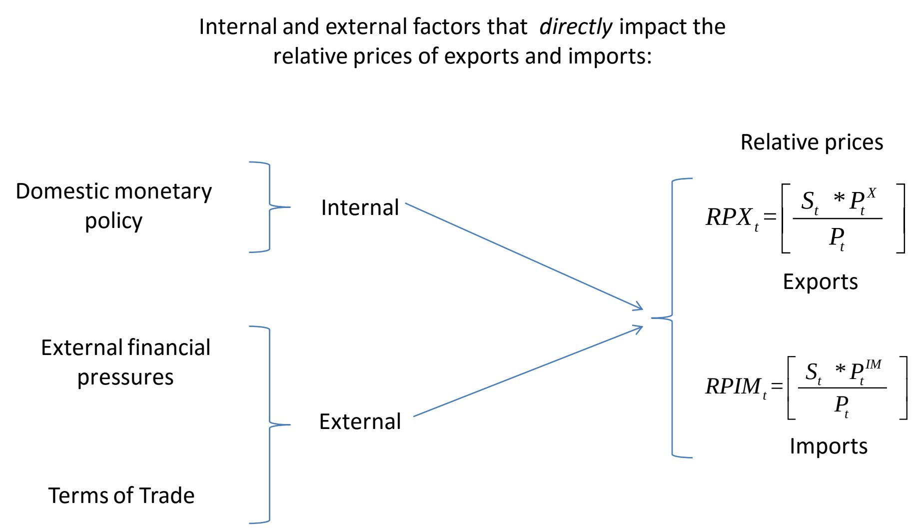
IV.a Exports, Imports, and the Trade Balance

The level of exports is assumed to comprise two elements: a long-run component which is expressed as a constant fraction of potential output and a short-run component which is linked to deviations of the relative price of exports from its long run norm:

$$X_t = Y^P[x + \eta_x * rpx_t] \tag{16}$$

where the share term x is determined by long-run external prices and productivity levels in the export and non-traded goods sectors, $\eta_x > 0$ is a response function and rpx_t is the percent deviation of the relative price of exports RPX_t from some long run norm.

Figure 3
Internal and external determinants of
relative prices of exports and imports



That is, the relative price of exports is defined as the world currency price of exports P_t^X converted to domestic currency by the nominal exchange rate S_t (domestic currency units per unit of foreign currency – appreciation minus), and divided by domestic price level, P_t:

$$RPX_t = \left[\frac{S_t \; * P_t^X}{P_t} \right] \tag{17}$$

We define a baseline value of RPX_t namely $\overline{RPX}$ which we normalize to unity. Thus, $rpx_t \approx \ln(RPX_t)$.

The corresponding equation for imports is structured much like that of exports but also includes a term for the output gap:

$$IM_t = Y_t^P [im + im_{cyc} * gap_t + \eta_{im} * rpim_t] \tag{18}$$

where the share term im is determined by long-run external prices and productive capacity of the economy, im_{cyc} is a short-run marginal propensity to import ($im_{cyc} > 0$), $\eta_{im} < 0$ is a response function and $rpim_t$ is the percent deviation of the relative price of imports $RPIM_t$ from some long run norm. Symmetric with exports, that price ratio is defined as:

$$RPIM_t = \left[\frac{S_t \; * P_t^{IM}}{P_t} \right] \tag{19}$$

Again, the baseline value $\overline{RPIM}$ is normalized to unity. Thus, $rpim_t \approx \ln(RPIM_t)$.

IV.b The Open Economy IS Curve

An open economy is vulnerable to shocks that originate externally. Such shocks will have first order impacts on exports and imports. The model highlights the fact that shocks – both domestic and external – are transmitted to the economy through their impacts on the relative prices of imports and exports – as summarized in Figure 3.

To see how we can model such impacts in an intuitive way, we first recall that both relative prices can be conveniently decomposed into a real exchange rate component and a weighted terms-of-trade component – the first two terms on the right hand side of equations (20) and (21):

$$\underbrace{rpx_t}_{\substack{\text{Relative price}\\ \text{of exports}}} = \underbrace{q_t}_{\text{Real exchange rate}} + \underbrace{(1-\nu)\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}} \qquad (20)$$

$$\underbrace{rpim_t}_{\substack{\text{Relative price}\\ \text{of imports}}} = \underbrace{q_t}_{\text{Real exchange rate}} - \underbrace{\nu\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}} \qquad (21)$$

where $q_t = \ln(S_t P_t^{EXT} / P_t) = \ln[S_t (P_t^X)^\nu (P_t^{IM})^{(1-\nu)} / P_t]$ is the logarithm of the real exchange rate whose baseline value normalized to unity, $TT_t = P_t^X / P_t^{IM}$ is the ratio of export prices to import prices in world markets whose baseline value $\overline{TT}$ is normalized to unity. (For further discussion of this decomposition, see Dabos and Juan-Ramón, 2000; see also Clarida, 2009, for a discussion of terms-of-trade impacts.)

Thus, the first elements on the right hand sides of (20) and (21) show that the relative prices of exports and imports each include two components: the real exchange rate and a scaled terms-of-trade component.

Next, this decomposition is extended to distinguish between domestic and externally based components. As discussed below, the real exchange rate is determined by a real interest rate parity condition; movements in the real exchange rate correspond one-to-one with movements in the real interest differential – a function of both domestic monetary policy and external financial conditions.

To establish a baseline for the model, the natural rate of interest is now assumed to be the sum of the natural external rate of interest $\bar{r}^{EXT}$ plus a baseline risk premium $\overline{rp}$. Importantly, the natural rate of interest is a steady-state construct. That is, the natural rate of interest is written $\bar{r} = \bar{r}^{EXT} + \overline{rp}$. That is, in any economy, open or closed, the steady state natural interest rate converges to the steady state marginal product of capital net of depreciation, namely $mpk^{SS} - \delta$. In an open economy, $mpk^{SS} - \delta$ converges to an exogenous value, namely $\bar{r}^{EXT} + \overline{rp}$ (thorough net capital accumulation). *This issues is discussed further below, in Part V.b.*

Regarding deviations from that steady state, tighter foreign monetary policy implies that $r_t^{EXT} > \bar{r}^{EXT}$. An idiosyncratic revision to investor perceptions of a country will be reflected its risk premium: a capital-flight scenario would imply that $rp_t > \overline{rp}$. Jointly, external financial pressures efp_t reflects the divergence between the external interest rate plus risk premium from their baseline values: $efp_t = [r_t^{EXT} + rp_t] - [\bar{r}^{EXT} + \overline{rp}]$.

We next introduce the real interest parity condition that tells us the short run deviation of the real exchange rate from its long-run baseline value: $q_t = r_t^{EXT} + rp_t - r_t + \bar{q}$, $\bar{q} = 0$. (That is, $\exp(\bar{q}) = 1$.)This condition implies impacts of domestic monetary policy and external financial pressures on the real exchange rate that are symmetric: a domestic monetary tightening causes the real exchange rate to appreciate; this reduces relative prices of both exports and imports. By contrast, an increase in external financial pressures will bring about a depreciation of the real exchange rate; this increases relative prices of both exports and imports.

Thus, relative prices of exports and imports are determined by both domestic and external factors is shown in an exact decomposition in the rightmost terms of equations (20') and (21'):

$$\underbrace{rpx_t}_{\substack{\text{Relative price}\\\text{of exports}}} = \underbrace{q_t}_{\text{Real exchange rate}} + \underbrace{(1-\nu)\ln(TT_t)}_{\substack{\text{Scaled External}\\\text{Terms of Trade}}} = \boxed{\underbrace{[\bar{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t + (1-\nu)\ln(TT_t)}_{\text{External shocks}}} \qquad (20')$$

$$\underbrace{rpim_t}_{\substack{\text{Relative price}\\\text{of imports}}} = \underbrace{q_t}_{\text{Real exchange rate}} - \underbrace{\nu\ln(TT_t)}_{\substack{\text{Scaled External}\\\text{Terms of Trade}}} = \boxed{\underbrace{[\bar{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t - \nu\ln(TT_t)}_{\text{External shocks}}} \qquad (21')$$

Domestic monetary policy has impacts on these relative prices through the real exchange rates. Externally, both external financial pressures and changes in the external terms-of-trade have impacts on these key relative prices.

Accordingly, export (supply) and import (demand) functions may be re-written as:

$$X_t = Y^P[x - \underbrace{\eta_x * (r_t - \bar{r})}_{\text{endogenous}} + \underbrace{\eta_x efp_t + \eta_x(1-\nu)\ln(TT_t)}_{\text{exogenous}}] \qquad (22)$$

$$IM_t = Y_t^P[im + \underbrace{im_{cyc} * gap_t - \eta_{im} * (r_t - \bar{r})}_{\text{endogenous}} + \underbrace{\eta_{im} efp_t - \eta_{im}\nu\ln(TT_t)}_{\text{exogenous}}] \qquad (23)$$

Both exports and imports will move when either when domestic monetary policy changes (an endogenous factor in the model), when there are externally-based financial pressures (exogenous) or when the external terms of trade change (exogenous).

We may now develop the open economy IS curve by substituting domestic expenditure functions (6b), (7) and (3) and external sector equations (22) and (23) into the open economy GDP identity (15) and rearranging:

$$gap_t \Big|_{IS,\text{Open}} = \frac{(\varphi_{Ir} - \eta_{nx})(r_t - \overline{r}) + fp_t + \eta_{nx} efp_t + \tilde{\eta}_{nx} \ln(TT_t)}{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}]\}} \qquad (24)$$

where the compressed fiscal policy component is $fp_t = gp_t - (1-\sigma_{cyc})tp_t$ while compound parameters are $\eta_{nx} = \eta_x - \eta_{im}$ and $\tilde{\eta}_{nx} = \eta_x(1-\nu) + \eta_{im}\nu$.

IV.c Monetary Policy and Inflation in the Open Economy

Regarding monetary policy in an open economy, since $\overline{r} = \overline{r}^{EXT} + \overline{rp}$, we add external financial pressures to the monetary reaction function:

$$i_t \Big|_{Open} = \overline{r} + \pi^e + b_\eta(\pi_t - \pi^*) + b_{gap} gap_t + efp_t + r_t^{DISC} \qquad (25)$$

The presence of efp_t in this equation confirms that, all else equal, the interest rate set by the central bank will track developments in world financial markets. Assuming (as a special case) that nominal exchange rate depreciation equals the inflation rate, if the inflation expectations gap, the output gap, and discretionary policy are all zero, equation (25) would be interpreted as a traditional "International Fisher" equation. As an alternative interpretation, by including efp_t in equation (25), the mandate of the central bank in an open economy is expanded compared to a closed economy. For example, $efp_t > 0$ in may be thought of as a capital outflow scenario. In this case, the central bank will take measures to defend the exchange rate by raising the interest rate. Such a reaction suggest that the central bank in an open economy is concerned about balance sheet effects of exchange rate movements – in addition to price and output stabilization. Such an idea is frequently discussed in the literature on emerging market macroeconomic policy; for example, see Calvo and Reinhart's (2002) discussion of 'fear of floating'. Note that, in *equilibrium* the interest rate will not move one-to-one with efp_t . Instead, an initial interest rate hike will be partially offset by an *induced* interest rate decrease which reflects the central banks goal to stabilize both prices and output on the downside.

Finally, inflation in the open economy differs from the closed economy expression insofar as a fraction θ of external financial pressures, which have impacts on the real exchange rate, area also passed through to the domestic economy:

$$\pi_t\Big|_{Open} = \pi^e + \frac{1}{\eta}(gap_t - ss_t) + \theta efp_t \qquad (26)$$

That is, the term θefp_t may be thought of as exchange rate pass-through that reflects external financial market developments.

By combining the open economy monetary policy rule (25) and the Phillips curve (26) we obtain the equilibrium real rate of interest in the open economy:

$$r_t\Big|_{RR,Open} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t - b_{RRss}ss_t + b_{RRefp}efp_t + r_t^{DISC} \qquad (27)$$

where the reduced form coefficient for external financial pressures is defined as $b_{RRefp} = [b_\pi - 1]\theta + 1$. This term will exceed unity: the central bank is assumed to raise interest rates both to defend the currency and to restrain exchange rate pass through to inflation.

IV.d Equilibrium in the Open Economy – Core Variables and the Trade Balance

To obtain equilibrium output in the small open economy, again flip IS curve (24) to obtain an expression for the real interest rate therein:

$$r_t\Big|_{IS,\text{Open}} = \bar{r} + \frac{\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}gap_t - [\,fp_t + \eta_{nx}efp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} \qquad (28)$$

We then solve for the gap by combining IS and RR curves for the open economy:

$$gap_t^{eq}\Big|_{Open} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + efp_t[b_{RRefp} + \frac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}] + \frac{[\,fp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} + r_t^{DISC}}{\left[\frac{\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}}{(\varphi_{Ir} - \eta_{nx})} - b_{RRgap}\right]} \qquad (29)$$

Note that the denominator on the right-hand side is negative. This must be so since: $\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}>0$, $(\varphi_{Ir} - \eta_{nx})<0$ and $b_{RRgap}>0$. We can therefore say that an increase in inflationary expectations relative to the target $(\pi^e - \pi^*)>0$ will reduce output, a favorable supply shock ss_t will increase output, while increases in domestic and foreign demand, fp_t and $\tilde{\eta}_{nx}\ln(TT_t)$ respectively, will each increase output. Importantly, an increase in external financial pressures will work through two channels that have opposite effects on output. The central bank's response to such pressures which are included in its reaction function will squeeze expenditures and reduce output. In the other direction, the currency depreciation effect, reflected in the term $[\eta_{nx}/(\varphi_{Ir}-\eta_{nx})]*efp_t$, will have a positive impact on output (a rightward shift of the IS curve). However, since the absolute value of $[\eta_{nx}/(\varphi_{Ir}-\eta_{nx})]$ must be less than one, the former effect dominates the latter. Finally, a discretionary interest rate hike will reduce output and cause the real exchange rate to appreciate.

As before, we obtain the two remaining core variables by substituting in gap_t^{eq} along with the other exogenous variables into equations (27) and (26) – the equilibrium real interest rate and inflation rate, r_t^{eq} and π_t^{eq} respectively. The equilibrium nominal interest rate is $i_t^{eq}=r_t^{eq}+\pi_t^{eq}$ -- consistent with equation (25). As in the case of the closed economy, we assume coherence between movements in the discretionary component of monetary policy and those of equilibrium values for interest rates (real and nominal): they are all in the same direction. To obtain the real exchange rate, we incorporate the equilibrium real interest rate and assumptions regarding external financial pressures into the parity equation $q_t = r_t^{EXT}+rp_t-r_t+\bar{q}$. And, as before, to obtain the expenditure side components in real currency units (Dollars), results for gap_t^{eq} and r_t^{eq} are inserted into equations (6b), (7), (3), (22) and (23), alongside the assumptions regarding tax and expenditure policy, gp_t and tp_t respectively, external financial pressures efp_t and foreign demand TT_t.

The equilibrium trade balance, in percent of potential, is a function of the equilibrium output gap and real exchange rate (conditional on terms-of-trade shocks):

$$nx_t^{eq} = x - im + (\eta_x - \eta_{im})q_t^{eq} + [\eta_x*(1-\nu)+\eta_{im}\nu]*\ln(TT_t)] - im_{cyc}*gap_t^{eq}$$

where the equilibrium real exchange rate is: $q_t^{eq} = r_t^{EXT}+rp_t-r_t^{eq}+\bar{q}$.

Examples of open economy calculations and their corresponding graphical exposition are presented in Figures 4-6. (The spreadsheet that generated the table and graphs is available online at ________________). The top section of Figure 4 shows assumed shocks for three scenarios. Under the baseline, there are no shocks. Under alternative scenario (i) there is an adverse shock to external demand – export prices fall by 4% from their baseline values. Alternative scenario (ii) retains that 4% export price decline but adds in a 400 basis point increase in the risk premium: efp_t =4%.

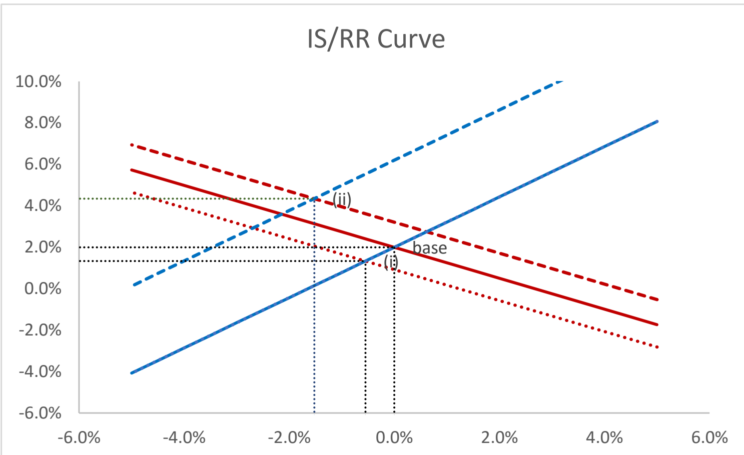
Figure 4
Small open economy model
Shocks and output gap components

		Shocks -- Expenditure		
In percent of potential output		base	alt(i)	alt(ii)
Gov't Spending	gp_t	0.0%	0.0%	0.0%
Tax Measures (one-off)	tp_t	0.0%	0.0%	0.0%
In percent		Shocks - Supply / Expected inflation		
Supply shock (% of Y^P)	ss_t	0.0%	0.0%	0.0%
Inflation expectations (gap w.r.t. target)	$\pi^e - \pi^*$	0.0%	0.0%	0.0%
In percent		Shocks - Discretionary monetary policy		
Deviation from Taylor Rule (shift)	r_t^{DISC}	0.0%	0.0%	0.0%
In percent		Shocks - External financial pressures (efp)		
Total external financial pressures		0.0%	0.0%	4.0%
Real interest rate -- dev.from baseline	$r_t^{EXT} - \bar{r}^{EXT}$	0.0%	0.0%	0.0%
Risk premium -- dev.from baseline	$rp_t - \overline{rp}$	0.0%	0.0%	4.0%
In percent		Shocks - External Terms of Trade (ln(TT))		
TT log deviation from baseline		0.0%	-4.0%	-4.0%
Export price log dev		0.0%	-4.0%	-4.0%
Import price log dev		0.0%	0.0%	0.0%

Calculation of equlibrium output gap -- component by component

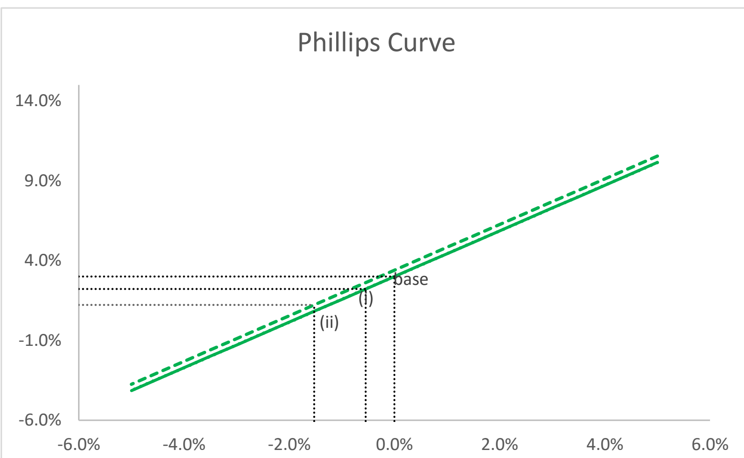
	base	alt(i)	alt(ii)
(a) Inflation expectations component: $\text{Inflation expectations component}=\frac{b_{RR\pi}(\pi^e-\pi^*)}{den}$	0.0%	0.0%	0.0%
(b) Supply shock component: $\text{Supply shock component} = \frac{b_{RRss} ss_t}{den}$	0.0%	0.0%	0.0%
(c) Fiscal component: $\text{Fiscal component} = \frac{\frac{fp_t}{(\varphi_{Ir}-\eta_{nx})}}{den}$	0.0%	0.0%	0.0%
(d) Discretionary monetary component: $\text{Discretionary monetary component} = \frac{r_t^{DISC}}{den}$	0.0%	0.0%	0.0%
(e) External financial pressure component: $\text{External financial pressures component} = \frac{efp_t[b_{RRefp} + \frac{\eta_{nx}}{(\varphi_{Ir}-\eta_{nx})}]}{den}$	0.00%	0.00%	-0.98%
(f) External terms of trade component: $\text{External terms of trade component} = \frac{\frac{\tilde{\eta}_{nx}\ln(TT_t)}{(\varphi_{Ir}-\eta_{nx})}}{den}$	0.00%	-0.55%	-0.55%

Figure 5
Small open economy Model:
Graphical exposition: core variables, net exports, real exchange rate



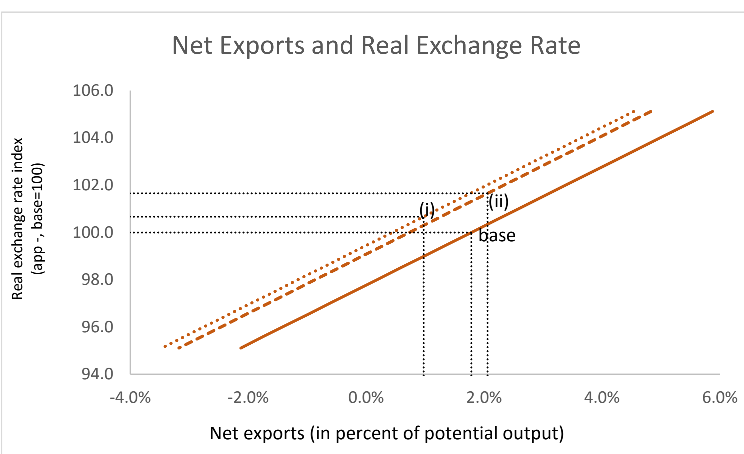
alt(i) – external terms of trade deterioration shifts IS curve to left (from solid to dotted) along solid RR curve.

alt(ii) – external financial pressures shift IS curve to right (dotted to dashed) but RR curve to the left (solid to dashed, exchange rate depreciation, interest rate defense)



alt(i) – external terms of trade deterioration reduces output gap and inflation – shift along original PC line.

alt(ii) – external financial pressures shift PC up and to the left (solid to dashed exchange rate depreciation passed through to inflation) but also further along dashed PC (output gap even lower).



alt(i) – external terms of trade deterioration dominates income effect, shifting NX curve to the left (solid to dotted, higher deficit); exchange rate depreciation implies upward shift *along* dotted tan NX line.

alt(ii) – external financial pressures imply lower output gap, rightward shift of tan NX line (from dotted to dashed, lower imports); further exchange rate depreciation means a further upward shift along dashed NX line.

Figure 6
Small open economy model:
Solutions for core variables, external sector, and expenditure components

Output gap (a)-(b)+(c)+(d)+(e)+(f)	0.0%	-0.5%	-1.5%

$$gap_t^{eq}\Big|_{Open} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + efp_t[b_{RRefp} + \frac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}] + \frac{[fp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} + r_t^{DISC}}{\left[\frac{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}]\}}{(\varphi_{Ir} - \eta_{nx})} - b_{RRgap}\right]}$$

Equilibrium inflation rate	3.0%	2.2%	1.2%

$$\pi_t\Big|_{Open} = \pi^e + \frac{1}{\eta}(gap_t^{eq} - ss_t) + \theta efp_t$$

Equilibrium real interest rate	2.0%	1.3%	4.3%

$$r_t\Big|_{RR,Open} = \overline{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t^{eq} - b_{RRss}ss_t + b_{RRefp}efp_t + r_t^{DISC}$$

Real price of exports (deviation from norm)	0.0%	-3.2%	-2.2%

$$\underset{\substack{\text{Relative price}\\\text{of exports}}}{rpx_t} = \underset{\text{Real exchange rate}}{q_t} + \underset{\substack{\text{Scaled External}\\\text{Terms of Trade}}}{(1-\nu)\ln(TT_t)} = \underbrace{[\overline{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t + (1-\nu)\ln(TT_t)}_{\text{External shocks}}$$

Real price of imports (deviation from norm)	0.0%	0.8%	1.8%

$$\underset{\substack{\text{Relative price}\\\text{of imports}}}{rpim_t} = \underset{\text{Real exchange rate}}{q_t} - \underset{\substack{\text{Scaled External}\\\text{Terms of Trade}}}{\nu\ln(TT_t)} = \underbrace{[\overline{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t - \nu\ln(TT_t)}_{\text{External shocks}}$$

Real exchange rate (deviation from norm) (appreciaton -)	0.0%	0.7%	1.7%
Real exchange rate index (Base = 100, app -)	100.0	100.7	101.7
Currency-unit results			
Gross Domestic Product	18.33	18.23	18.05
Consumption	11.92	11.89	11.85
Investment	3.34	3.41	3.08
Governerment Spending	2.75	2.75	2.75
Net Exports	0.33	0.18	0.38
Exports	1.45	1.21	1.29
Imports	1.12	1.04	0.91
Net Exports/Y^P	1.8%	1.0%	2.1%
Net Exports/Y^P Baseline	1.8%	1.8%	1.8%
NX gap	0.0%	0.8%	-0.3%
Demand side decomposition of output gap (percent of potential)			
Output gap	0.0%	-0.5%	-1.5%
Consumption	0.0%	-0.1%	-0.4%
Investment	0.0%	0.4%	-1.4%
Governerment Spending	0.0%	0.0%	0.0%
Net Exports	0.0%	-0.8%	0.3%
Exports	0.0%	-1.3%	-0.9%
Imports	0.0%	-0.5%	-1.2%

The impacts of these shocks on the output gap are shown in the lower portion of the figure – component-by-component. Under both alt(i) and alt(ii), the terms of trade shock reduces output by 0.55%. Under alt(ii), adverse external financial pressures further reduce output by 0.98%.

The graphs in Figure 5 illustrate the channels of transmission for each of these shocks. By itself, the terms-of-trade shock in alt(i) shifts the IS curve (red line, uppermost chart) to the left – from solid to dotted. The central bank, consistent with its mandate and keeping on its Taylor Rule, reduces the interest rate – a movement along the RR curve downward and to the left. Thus, the initial impact of the adverse demand shock is softened by lower interest rates.

Unsurprisingly, there is lower inflation under scenario (i) – as reflected in a leftward movement along the solid green Phillips Curve line in the middle chart. The corresponding numerical solution, as shown in figure 8, implies a fall in the equilibrium inflation rate from its initial target of 3% to 2.2%.

It is also not surprising that the net exports deteriorate but the real exchange rate depreciates under scenario (i). This is shown graphically in the bottommost chart of figure 6. Here, net exports as a ratio to potential output are shown on the horizontal axis (leftward movement means higher deficit) while the real exchange rate is on the vertical axis (upward movement means currency depreciation).

Thus, the tan line reflects the relationship between the real exchange rate and net exports for *given values of the output gap and the terms of trade*. That line slopes upward: when the currency depreciates, exports are encouraged, imports are discouraged, and the trade balance improves. An increase in the output gap means more imports (an income effect); this means that the curve will shift to the left. A reduction in the terms of trade means lower external demand – again a leftward shift of the tan line. Of course, a reduction of the terms of trade also reduces the output gap: demand decreases. This is reflected as second order effect -- a partially offsetting shift of the tan line the right. However, the initial impact typically dominates: a fall in the terms of trade will mean a net shift of the tan line to the left.

Such an impact is shown for alternative (i). The terms-of-trade impact, which dominates the output impact, shifts the net export line to the left from the solid tan NX line to the dotted line: any given level of the real exchange rate, the deficit will now be higher. As an offsetting factor, the depreciation of the real exchange rate – the outcome of a monetary loosening – brings about an upward shift *along* the dotted tan line.

What happens to inflation under alternative (ii)? Adverse external financial pressures cause the currency to depreciate in real terms – higher import prices. To some extent, these higher costs will be passed on to domestic purchasers – as the rightmost term in equation (26) (the open economy Phillips Curve) reveals. For this reason, the green line shifts up and to the left

– from the solid to the dashed line. At the same time, the equilibrium output gap is even lower under alt(ii) than alt(i) – as reflected in a shift *along* that green dashed line. The calculation in figure 8 confirms this: equilibrium inflation is now 1.2% (compared to 2.2% under alt(i)).

What happens to net exports under alternative (ii)? Because higher real interest rates at home further squeeze demand, equilibrium output is even lower under alt(ii) than alt(i). Import compression under alt(ii) means that the tan NX line now shifts to the right – from the dotted to the dashed line. At the same time, the real exchange rate depreciates more under alt(ii) than alt(i). (Without the central bank's defensive interest rate hike under alt(ii), the depreciation would have been even more severe.) This means a further shift along the dashed tan NX line.

Together, income and price effects bring about an improvement in the net export balance under alt(ii) relative to alt(i). But the net export improvement under alt(ii) should be interpreted as a *forced adjustment* – a capital outflow scenario that is accompanied by an even sharper reduction of output under alt(ii) than under alt(i).

The numbers in the bottom section of Figure 6 confirm the graphical analysis. In addition, the demand side contributions at the bottom of that figure show that, under a capital outflow scenario, the burden of adjustment falls mainly in investment (minus 1.5% of potential output) and also to some extent on consumption (minus 0.4% of potential output). This is consistent with Calvo's (1998) analysis of *sudden stops* in the financial account of the balance of payments: abrupt reversals of capital inflows will be reflected in an increase in the current account balance along with a cutback of domestic expenditures.

IV.e The trade balance and monetary policy: In the Spirit of Marshall and Lerner

A question arises: what is the impact of a discretionary shift in monetary policy on the trade balance? Since the movement of the discretionary component of monetary policy and the equilibrium real interest rate itself (including induced movements) are assumed to be in the same direction, and there are no other shocks, the real exchange rate must depreciate when there is a discretionary monetary loosening $r_t^{DISC}<0$. That is, assuming no other shocks, the impact of discretionary monetary policy on the equilibrium real interest rate is:

$$r_t^{eq}\Big|_{RR,Open} = \overline{r} + [\frac{b_{RRgap}}{den} + 1] * r_t^{DISC}$$

where den is the denominator in equation (29):

$$den = \left[\frac{\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}}{(\varphi_{Ir}-\eta_{nx})} - b_{RRgap} \right] < 0$$

That is, the full impact of discretionary monetary policy on the equilibrium real interest rate includes a second-order (or induced) change (sign opposite to r_t^{DISC}). Recall that coherence between discretionary policy and the equilibrium real interest rate implies that $[\frac{b_{RRgap}}{den}+1]>0$. This ensures that the equilibrium real interest rate will move in the same direction as the discretionary element.

Correspondingly, the full impact of discretionary monetary policy on the real exchange rate, assuming *no other shocks* and that the future real exchange rate $\bar{q}$ equals zero, can be seen as the sum of the direct impact plus the second-order (or induced) change:

$$q_t^{eq} = \bar{r} - r_t^{eq} = -[\frac{b_{RRgap}}{den}+1] * r_t^{DISC}$$

We may now sign the effect of monetary policy shocks to the trade balance. Since we have assumed that the real exchange rate is domestic currency per unit of foreign currency (depreciation plus), $(\eta_x - \eta_{im}) > 0$. We may think of this as a Marshall-Lerner condition, but in a narrow sense: a depreciation of the currency, ignoring all other effects, causes the trade balance to improve.

Alone, the coherence property discussed above ensures that monetary loosening will cause the real exchange rate to depreciate. However, coherence alone is not sufficient to ensure that the trade balance improves when there is a monetary loosening. Rather, the output gap effect on imports must be considered. This impact is written:

$$im_{cyc} * gap_t^{eq} = im_{cyc} * \frac{r_t^{DISC}}{den}$$

Thus, assuming no other shocks, the full impact of discretionary monetary policy on the trade balance is written:

$$(x_t^{eq} - im_t^{eq}) - (x - im) = r_t^{DISC} * [(\eta_x - \eta_{im}) * \{-[\frac{b_{RRgap}}{den}+1]\} - \frac{im_{cyc}}{den}]$$

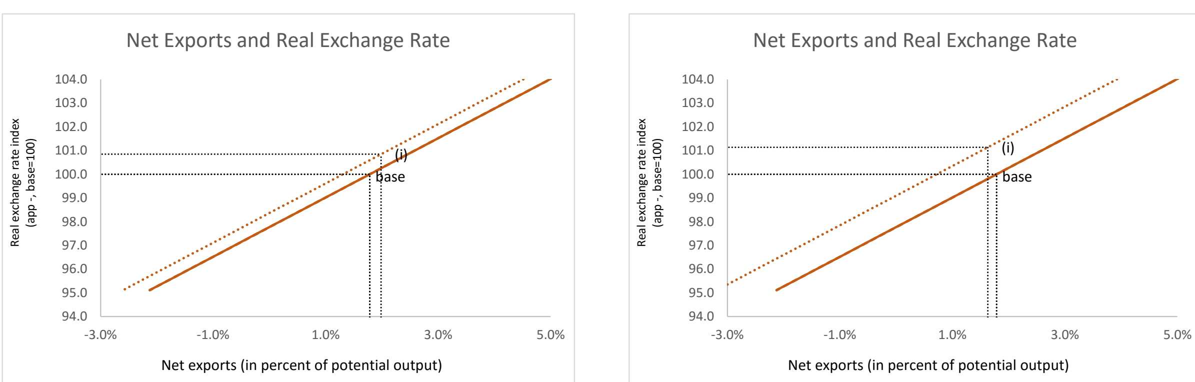
This permits us to consider an expanded Marshall-Lerner (EML) condition – one that also includes the impact of changes in the output gap on imports. A monetary loosening will cause the trade balance to improve if:

$$[(\eta_x - \eta_{im})*\{-[\frac{b_{RRgap}}{den}+1]\} - \frac{im_{cyc}}{den}] > 0$$

That is, the EML condition implies that the improvement in the trade balance that takes place purely through price changes (the real exchange rate) exceeds any deterioration thereof that reflects the impact of the output gap effect on imports.

Figure 7 presents a graphical interpretation of the EML condition. In the left-hand panel, alternative scenario (i) shows the effect of a discretionary monetary loosening (r^{DISC} <0) compared to the baseline when the Expanded Marshall-Lerner (EML) condition holds. The depreciation of the real exchange rate, which is reflected as an upward shift along the tan lines dominates the output gap effect which +is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is satisfied, a monetary loosening causes net exports to increase. In the right-hand panel, the alternative scenario shows the effect of a that same monetary loosening when the Expanded Marshall-Lerner (EML) condition fails to holds The depreciation of the real exchange rate, which is reflected as an upward shift along the tan lines is dominated by the output gap effect which +is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is not satisfied, a monetary loosening causes net exports to decrease.

Figure 7
The expanded Marshall-Lerner (EML) condition.



Alt (i) shows the effect of a discretionary monetary loosening r^{DISC} =-2% compared to the baseline. when the Expanded Marshall-Lerner (EML) condition holds. The depreciation of the real exchange rate, which is reflected as an upward shift along the tan lines dominates the output gap effect which +is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is satisfied, a monetary loosening causes net exports to increase.

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V. Long-run expenditure shares: Some links to microeconomics

In order to keep the preceding models simple, little effort has been made to derive the parameters from the explicit maximization of an objective function. However, it is not necessary to entirely jettison links to microeconomic models. There are instances where we might draw informal or heuristic linkages with such models.

One such instance regards the values of the long-run expenditure shares – steady-state ratios of consumption, investment, government, and net exports to potential output. In this case, it is both possible and helpful to our intuition to appeal to some microeconomics when deriving these values. Since the reasoning for the closed and open economy differ from one another somewhat, we present them as distinct analytical cases. *However, it can be seen that, in both cases, the long-run expenditure shares can be linked to the underlying 'deep parameters' of preferences and the production function.*

V.a The Closed Economy

Output is comprised of a private (after tax) component plus government expenditures:

$$Y = AK^{\alpha}L^{1-\alpha} = \underbrace{(1-\tau)AK^{\alpha}L^{1-\alpha}}_{\textit{Private output}} + \underbrace{G}_{\substack{\textit{Government} \\ \textit{Consumption}}} \tag{30}$$

There is one representative consumer / worker in the private sector: L is thus normalized to unity. Lifetime (logarithmic) utility for that individual is:

$$U(C) = \ln(C_t) + \frac{\ln(C_{t+1})}{(1+\rho)} + \frac{\ln(C_{t+2})}{(1+\rho)^2} + ... \tag{31}$$

where $1+\rho$ is the gross rate of time preference. In this Robinson Crusoe setting, the opportunity cost of current private consumption must be the gross after-tax marginal product of capital net of depreciation, $1+(1-\tau)\alpha AK^{\alpha-1}-\delta$. Thus, to satisfy the familiar Euler equation in steady state ($C_t = C_{t+1} = \overline{C}$), that marginal productivity term must equal the subjective rate of time preference:

$$(1-\tau)\alpha AK^{\alpha-1} - \delta = \rho \tag{32}$$

Rearranging this identity permits us to solve for the steady state stock of capital:

$$K^{SS} = \left[\frac{\rho + \delta}{(1-\tau)\alpha A} \right]^{\frac{1}{\alpha - 1}} \tag{33}$$

In steady state, output is thus:

$$Y^{SS} = A\left[\frac{\rho + \delta}{(1-\tau)\alpha A} \right]^{\frac{\alpha}{\alpha - 1}} \tag{34}$$

The steady state gross investment share must be the depreciation rate multiplied times the steady state capital/output ratio:

$$\psi = \delta \left[\frac{K}{Y} \right]^{SS} = \frac{\delta}{A}\left[\frac{\rho + \delta}{(1-\tau)\alpha A} \right]^{\frac{1}{\alpha}} \tag{35}$$

Finally, the net consumption ratio must be:

$$(1-\tilde{\sigma}) = 1 - \psi - \tau \tag{36}$$

That is, private savings provides financing for private investment plus the government expenditures (output equals consumption plus savings plus tax revenue).

V.b The Small Open Economy

In extending the model to a small open economy, there are certain analytical issues that need to be revisited. Importantly, the natural real interest rate is exogenously determined in world markets. It is assumed to be the sum of baseline values for global external rate of interest and a risk premium. We need to show conditions under which this interest rate is consistent with output at the steady state level – a zero output gap – assuming that all other variables are at their baseline values (i.e. no shocks).

To generate such a result, two conditions much be satisfied: (i) the capital stock is at its steady state level and (ii) the allocation of employment across sectors is consistent with equalized wages. Below, we develop a model of a small open economy that produces two goods: non-tradables (*N*) and exports (*X*). In this model, we show how both conditions will be satisfied.

The economy-wide production function is:

$$\tilde{Y} = Y_N + P_X Y_X = A_N K^{\alpha} L_N^{1-\alpha} + P_X A_X K^{\alpha} L_X^{1-\alpha} \tag{37}$$

where A_N and A_X are respectively, total factor productivities in non-traded and export sectors, K is the total capital stocks deployed to these two sectors, and L_N and L_X are the employment levels in each sector, and α is capital's share of total output. The labor force is assumed to be fixed: $L = L_N + L_X$. The price of non-tradables P_N is normalized to unity. To ensure a tractable solution, we assume that capital services are non-rival across sectors – i.e. "infrastructure."

In a steady state, the level and composition of output must satisfy two conditions: (i) the capital stock must be at its steady-state levels; (ii) the allocation of employment across sectors is consistent with equalized wages.

To satisfy condition (i), recall first that in the open economy, the natural rate of interest is exogenous: $\overline{r} = \overline{r}^{EXT} + \overline{rp}$. In the steady state, this interest rate must equal the after tax marginal product of capital net of depreciation. Thus, the steady-state first order condition for capital accumulation is:

$$\alpha(1-\tau)(K^{\alpha-1})*[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}] - \delta = \overline{r} \tag{38}$$

Then, the steady-state capital stock is:

$$K^{SS} = \left[\frac{\overline{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]}\right]^{\frac{1}{\alpha-1}} \tag{39}$$

Accordingly, steady-state output must be:

$$Y^{SS} = \left[\frac{\overline{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]}\right]^{\frac{\alpha}{\alpha-1}} * \left[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}\right] \tag{40}$$

To satisfy condition (ii) note that wage equality means that the value of marginal products of labor must be equalized across sectors. Thus the intersectoral allocation of labor must be:

$$\left[\frac{L_N}{(L_N+L_X)}\right]^* = \left\{1+\left[\frac{P_X A_X}{A_N}\right]^{\frac{-1}{\alpha}}\right\} \equiv \omega \tag{41}$$

That is, if wages are equalized, a fraction of the total labor force ω will be deployed in the non-traded goods sector while $(1-\omega)$ will be deployed in the export sector. In steady-state, export production is:

$$[P_X X]^{SS} = \left[\frac{\bar{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]} \right]^{\frac{\alpha}{\alpha-1}} * \left[P_X A_X L_X^{1-\alpha} \right] \tag{42}$$

Then, using (5) and (6) above, the steady-state share of exports (ratio to potential output) must be:

$$x = \frac{P_X A_X \left([1-\omega]L\right)^{1-\alpha}}{\left[A_N \left(\omega L\right)^{1-\alpha} + P_X A_X \left([1-\omega]L\right)^{1-\alpha} \right]} \tag{43}$$

We assume that in the steady state, the country receives constant external transfer Z^{SS} which is minus one times net exports:

$$Z^{SS} \equiv zY^{SS} \equiv im - x \equiv -nx \tag{44}$$

The volume of gross investment expenditures corresponds entirely to maintenance (constant capital stock):

$$\psi = \delta \left[\frac{K}{Y} \right]^{SS} = \delta \left[\frac{\bar{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]} \right]^{\frac{1}{\alpha}} \left[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha} \right] \tag{45}$$

The share of government expenditures must match the tax rate: $\gamma = \tau$. Finally, private consumption may be computed as a residual:

$$c = 1 - \psi - \gamma + z \tag{46}$$

VI. Extension: Logarithms, Growth Rates, and Elasticities

Macroeconomists often write behavioral relationships in terms of growth rates rather than levels. Parameters in their models take the familiar from of an elasticity rather than level response parameters (such as those introduced above).

As an example, an econometrician may estimate a consumption function as:

$$\ln(C_t) = \eta_{C0} + \eta_{CY} * \ln(Y_t^d) \tag{47a}$$

Using the approximation that $\Delta \ln(X_t) \approx \%\Delta X_t$ we may obtain an expression for the growth rate of consumption, namely:

$$\%\Delta C_t = \eta_{CY} \%\Delta Y_t^d \tag{47b}$$

Consistent with equation (47b), consumption in the current period is:

$$C_t = C_{t-1} + \eta_{CY} \frac{C_{t-1}}{Y_{t-1}} \Delta Y_t^d$$

That is, we simply re-write the short-run marginal propensity to consume from equation (1a) as $a_{CY} \equiv \eta_{CY} C_{t-1}/Y_{t-1}$.

Correspondingly, an econometric estimate for the investment function might be:

$$\ln(I_t) = \eta_{I0} + \eta_{Ir} * r_t \tag{48a}$$

where η_{Ir} is the familiar semi-elasticity of investment with respect to the real interest rate. Again using the approximation for growth rates, we have:

$$\%\Delta I_t = \eta_{Ir} * \Delta r_t \tag{48b}$$

Thus,

$$I_t = I_{t-1}[1 + \eta_{Ir} \Delta r_t] \tag{48c}$$

Here, we have simply re-written the interest rate response parameter in equation (2) as: $a_{Ir} \equiv \eta_{Ir} I_{t-1}$.

If we assume that that baseline values potential output and the natural rate of interest remain constant, we can use these expressions to generate scenarios based on alternative assumptions for short-run fiscal, monetary, and external shocks for a given period.

VII. Conclusion: Mind the algebra – even with 'simple' models.

In recent years, macroeconomic models have become increasingly complex. Such models do an important job: they help us understand the behavior of macroeconomic aggregates in a way that reflects rational, optimizing behavior by agents in that economy.

But, even with this progress, models that are simpler – the traditional IS/LM and its heirs – have not vanished from use. Both Krugman (2000) and Blanchard (2009) suggest that simpler models can play a useful role in day-to-day discussions by policy-oriented economists. And, such models continue to be the preferred pedagogical device for novices – including undergraduate students in economics.

One of the traditional features of these models is that comparative statics can easily be displayed graphically. Many important insights can be derived by shifting the familiar IS and monetary policy (LM or RR) curve. However, this paper has suggested that we can move beyond the graphics and obtain explicit calculations for key macroeconomic variables that are consistent with the graphs. We might think of such calculations as an extended 'back-of-the-envelope' exercise that might be used as a cross-check for more complex ones.

At this point, we may ask just what makes a model 'simple?' In this vein, 'simplicity' does *not* imply that we abandon algebra. But, we might think of a 'simple' model as one that can be solved using secondary-school algebra (rather than an iterative solution or more advanced techniques). We might think also think of a 'simple' model as one that relies on reduced form (slope) parameters – much as elementary demand and supply functions do.

Rather than presenting a new model per se, this paper has attempted to restate what might be thought of as models that are traditional and familiar. But, these models are presented in different algebraic spaces. As a technical matter, we can easily reconcile two equations or systems that are expressed in different units. The trick is to do so in a way that "…helps intuition and communication," as Blanchard (2009) suggested. In this sense, a key message of this paper is that we have to pay close attention to the algebra of models – even if those models are inherently simple.

As an initial example of such an algebraic bridge, this paper proposed to recast the traditional currency-unit expenditure equations – consumption, investment, government spending -- in terms of potential output. In this way, we are able easily transform an IS curve from currency unit space to output gap space – the metric of modern New Keynesian models that used both a Phillips Curve and a Taylor-type monetary reaction function. An example of both the graphical exposition and the back-of-the envelope calculation was presented in Figures (1) and (2).

Do the equations of this form help our intuition and our communication? Admittedly, such equations may superficially appear to be complicated. But, the algebraic scaling does provide more insight into deeper economic issues. For example, the constant term in consumption function (6b) links directly back to a model of long-run growth – for example the Solow model – which textbooks often feature as an antecedent to the short-run model. Likewise, an equation such as (6b) can help illustrate other key ideas in macroeconomics, such as the permanent income/life cycle approach and the Ricardian equivalence hypothesis, more readily than its traditional counterpart. [6]

The ideas are easily extended to an open economy. One of the key ideas in the paper is that external shocks in both goods and financial markets can be conveniently summarized in identities that link the relative prices of exports and imports to the real exchange rate and the external terms of trade. An advantage of this approach is that we rely to a large extent on an identity to understand how external factors can shift the IS and monetary policy curves. The model is especially helpful to understand the macroeconomic implications of shocks to the terms of trade and to external financial flows (i.e. 'sudden stops'). An application of the open economy model to such issues was shown in Figures 4-6.

Even though our short-run macro model has not been explicitly derived from a microfounded model, linkages to microeconomic foundations should not be jettisoned entirely. Examples of how to derive the long-run expenditure shares from taste and technology parameters were shown in Section V. And as a final algebraic bridge, Section VI shows how to reconcile a New Keynesian model with expenditure equations that are written terms of natural logarithms or percentage changes. In this case, the parameters of the New Keynesian may be recast to reflect the values of more familiar elasticities.

In sum, 'simple' models can serve a supplement to their more sophisticated counterparts. By paying some attention to algebra that links up different metrics – the algebraic 'Babel fish' referred to earlier -- we are able to get our simple models add considerable insight. Spreadsheet based example of the approaches herein are available online at ______.

Appendix: The 'Macronia' model – a financial programming exercise.

An online course on Financial Programming and Policies (FPP2x) that was recently developed by the International Monetary Fund's Institute for Capacity Development (ICD)

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[6] To see how such ideas might be presented to undergraduates, see the author's textbook manuscript (2014) currently available online at: http://testmodel2014.yolasite.com/. For example, the issue of household consumption is addressed in Chapter 7.

includes a model for a hypothetical case study ("Macronia") that is similar to the open economy model presented in the main body of the paper. For the real sector, participants are given equations in the more traditional 'elasticities' like those shown in Section VI. Using these equations, they build a real sector – element by element. The model, whose parameters are calibrated to match the each sector's equations in section VI, is used to both assess the economy's vulnerabilities and to design an adjustment program.

However, note that several additional features have been included into that model. First, output appears as a determinant of investment – an accelerator function. Thus, the equation is now written:

$$I_t = \tilde{a}_{I0} + a_{IY} Y_t + a_{Ir} r_t \tag{A1}$$

The corresponding expression written in terms of potential output and the output gap is:

$$I_t = Y^P [\psi + \varphi_{Icyc} gap_t + \varphi_{Ir}(r_t - \overline{r})] \tag{A2}$$

Thus, in equation (A2), the constant term would now be interpreted as:

$$\tilde{a}_{I0} = Y^P [\psi - \varphi_{Icyc} - \varphi_{Ir} \overline{r}]$$

Thus, the IS curve is now written as:

$$gap_t \Big|_{IS,\text{Open}} = \frac{(\varphi_{Ir} + \eta_{nx})(r_t - \overline{r}) + fp_t + \eta_{nx} efp_t + \tilde{\eta}_{nx} \ln(TT_t)}{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}] - \varphi_{Icyc}\}} \tag{A3}$$

Second, interest rate smoothing has been added to the central bank's interest rate rule. In nominal terms, we have:

$$i_t \Big|_{Open} = \rho i_{t-1} + (1-\rho)[\overline{r} + \pi^e + b_\eta(\pi_t - \pi^*) + b_{gap} gap_t + efp_t] + r_t^{DISC} \tag{A4}$$

Where the continuous smoothing parameter is $0 < \rho < 1$ ($\rho = 0$ implies no smoothing). The equilibrium real interest rate (Phillips curve substituted into Taylor rule) yields:

$$\begin{aligned} r_t \Big|_{RR,Open} = {} & \rho i_{t-1} + (1-\rho)[\overline{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap} gap_t - b_{RRss} ss_t + b_{RRefp} efp_t] \\ & - [\pi^e + \frac{1}{\eta}(gap_t - ss_t) + \theta efp_t] + r_t^{DISC} \end{aligned} \tag{A5a}$$

or more compactly:

$$r_t\big|_{RR,Open} = \bar{r} + \tilde{b}_{RR\pi}(\pi^e - \pi^*) + \tilde{b}_{RRgap}gap_t - \tilde{b}_{RRss}ss_t + \tilde{b}_{RRefp}efp_t + \tilde{r}_t^{DISC} \qquad \text{(A5b)}$$

where $\tilde{b}_{RR\pi} = (1-\rho)b_{RR\pi}$, $\tilde{b}_{RRgap} = (1-\rho)b_{RRgap} - \dfrac{\rho}{\eta}$, $\tilde{b}_{RRss} = (1-\rho)b_{RRss} - \dfrac{\rho}{\eta}$,

$\tilde{b}_{RRefp} = (1-\rho)b_{RRefp} - \dfrac{\rho}{\theta}$ are compound coefficients and $\tilde{r}_t^{DISC} = r_t^{DISC} + \rho(i_{t-1} - \pi^e - \bar{r})$ is a compound error term that includes both the contemporaneous discretionary element and the smoothing component $\rho(i_{t-1} - \pi^e - \bar{r})$.

Thus, adding these two elements, the equilibrium output gap is now:

$$gap_t^{eq}\big|_{Open} = \frac{\tilde{b}_{RR\pi}(\pi^e - \pi^*) - \tilde{b}_{RRss}ss_t + efp_t[\tilde{b}_{RRefp} + \dfrac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}] + \dfrac{[\,fp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} + \tilde{r}_t^{DISC}}{\left[\dfrac{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}] - \varphi_{Icyc}\}}{(\varphi_{Ir} - \eta_{nx})} - \tilde{b}_{RRgap}\right]} \qquad \text{(A6)}$$

Examples of this solution are found in the ICD’s online financial programming course FPP2x, applied to the case of “Macronia.”

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