Fiscal and External Shocks under a Soft Peg Regime:
A Practical Tool for Policy Makers

Evan C. Tanner
etanner@imf.org
Institute for Capacity Development/Asian Division
International Monetary Fund
Washington, DC 20431

May 2017
DRAFT – Do not cite

ABSTRACT: This note presents a simple model of a small open economy with a fixed exchange rate. The model is designed to show how both domestic and external shocks are transmitted to core macroeconomic variables. Much like traditional Keynesian frameworks, this model yields comparative static results that are easily displayed in a graphical format. In addition, the model also provides corresponding estimates for core macroeconomic variables and expenditure components – including the trade balance. And, by extension, the model permits linkages between government financing and reserve flows. In this sense, the model is meant to be used as a *practical* tool for policy makers in small economies that have adopted a soft-peg. To this end, a small spreadsheet based model is available as a companion to the paper. The spreadsheet is available on http://www.evanctanner.com/simple-models.

1. *Introduction*

The soft-peg exchange rate regime has not disappeared. Quite to the contrary, Rose (2011) documents that approximately half of all countries in the world have adopted some sort of fixed exchange rate regime; of those, most countries are on an adjustable or 'soft' peg. Even so, most recent research papers in macroeconomics – including those in a 'New Keynesian' vein – examine the countries with independent monetary policies which permit the exchange rate to fluctuate.

Most countries with fixed exchange rates are small relative to the world economy. Many are vulnerable to exogenous shocks – including those from global markets. For planning purposes, policy makers need to make assessments of how exogenous shocks will impact the core macroeconomic variables. Moreover, in order to maintain that fixed peg, policy makers need to keep a watchful eye on the central bank's holdings of international reserves.

This note presents a simple model of a small open economy with a fixed exchange rate. The model is designed to show how both domestic and external shocks are transmitted to core macroeconomic variables. Much like traditional Keynesian frameworks, this model yields comparative static results that are easily displayed in a graphical format. In addition, the model also provides corresponding estimates for core macroeconomic variables and expenditure components – including the trade balance. And, by extension, the model helps illustrate linkages between government financing and reserve flows.

The model extends one that was developed for flexible exchange rate economies by Tanner (2017). The model is intentionally 'simple' in the spirit of Krugman (2000), Romer (2000), Walsh (2002), and Blanchard (2009). Its theoretical underpinnings are close to traditional Keynesian models. In this sense, the model discussed herein can be used as a teaching tool. At the same time, since the model also provides quantitative estimates, it may also be used as a *practical* tool for policy makers in small economies that have adopted a soft-peg.

2. Domestic Expenditures: Some Familiar Equations

The model in this paper features an expression for the output gap which is built "bottom up" from the components of expenditure. The steps required to obtain such an expression is more fully detailed in Tanner (2015). Following the steps in that paper, we will first consider expressions for domestic expenditure that are expressed in a familiar metric: real units of the domestic currency. Thus, to begin, consider a standard (Keynesian) function for consumption C_t :

$$C_t = a_{C0} + a_{CY}(Y_t - T_t) \tag{1a}$$

where Y_t denotes output T_t denotes tax revenue taken by the government and a_{CY} is the marginal propensity to consume out of disposable income $Y_t - T_t$, $0 < a_{CY} < 1$.

We assume that tax revenue itself is a constant fraction of output τ plus a temporary lump-sum component "tax policy" component TP_t :

$$T_t = \tau Y_t + TP_t \tag{1b}$$

Thus the consumption function is now:

$$C_t = a_{C0} + a_{CY}(Y_t(1-\tau) - TP_t) \tag{1c}$$

Next, the level of investment I_t is a function the real interest rate r_t:

$$I_t = a_{I0} + a_{Ir} r_t \tag{2}$$

where $a_{Ir} < 0$.

Finally, government expenditure is some normal level (a constant), namely a_{G0} plus a "government policy" shock, GP_t :

$$G_t = a_{G0} + GP_t \tag{3}$$

In this example we assume that government spending is exclusively (not capital) spending. However, that assumption is not essential and may be modified in an extension of the model.

3. Rescaling domestic expenditures equations in terms of potential output

Equations (1-3) should be quite familiar: they are similar to those that have appeared in many macroeconomics textbooks over the years. Also, they are written in the most familiar metric: domestic currency units. However, since our goal is to express the output gap as the sum of demand-side elements, each of these equations must be expressed in a way that distinguishes their long-run or structural elements from their short-run or transitory components – including those linked to the output gap.

To obtain such a rescaling, we first assume some level of potential output in any period Y^P. Second, for the household consumption equation, we assume a constant tax rate τ which is applied to both potential output and the output gap $Y - Y^P$. Thus, we add and subtract τY^P from (1b) to obtain:

$$T_t = \tau Y^P + \tau(Y_t - Y^P) + TP_t \qquad (4)$$

We now see that tax revenues have three components: a long run or structural component τY^P, a cyclical component $\tau(Y_t - Y^P)$ and a short-run policy measure or "one-off" component TP_t.[1] Next, we re-write the consumption function itself. We first add and subtract the term $a_{CY}(1-\tau)Y^P$ from the right-hand side of consumption function (1c) and rearrange to obtain:

$$C_t = \tilde{a}_{C0} + a_{CY}\{[(1-\tau)(Y_t - Y^P)] - TP_t\} \qquad (5a)$$

where $\tilde{a}_{C0} = a_{C0} + a_{CY}(1-\tau)Y^P$. It will be useful to redefine this constant term in the equation as: $\tilde{a}_{C0} = Y^P(1-\sigma)*(1-\tau)$ where is the economy's long-run rate if saving rate. Thus, consumption function (5a) is now reinterpreted as:

$$C_t = \underbrace{(1-\tilde{\sigma})Y^P}_{\text{long-run component}} + \underbrace{(1-\sigma_{cyc})\{[(1-\tau)(Y_t - Y^P)] - TP_t\}}_{\text{short-run component}} \qquad (5b)$$

where $(1-\tilde{\sigma}) = (1-\sigma)*(1-\tau)$. The interpretational advantages of (5b) over (1c) are discussed more fully in Tanner (2015). In both equations, the first term on the right-hand side is a constant. In the rescaled equation (5b), that constant *explicitly* informs us about household consumption and disposable income in the long run. The parameter $\tilde{\sigma}$ may be thought of as the long-run savings rate (adjusted for taxes). Such a parameter can be clearly traced back to a long-run growth model that includes taxes -- either the Solow model or (as discussed in greater detail in Tanner, 2015) one based on intertemporal optimization.

---

[1] This simple version of the model does not include automatic stabilizers. To do so is easy: we just assume a cyclical tax rate τ_{cyc} that is negative. In this case, the second term in equation (4) becomes $\tau_{cyc}(Y_t - Y^P)$ whose positive value when output is below potential tells us the size of government safety-net transfers.

We may also rescale the investment function (2). We assume that there is a natural (or neutral) real rate of interest $\bar{r}$ which, in the absence of any other shocks to the economy, yields a zero output gap. We subtract and add $a_{Ir}\bar{r}$ to the right-hand side of that equation and rearrange to obtain:

$$I_t = Y^P[\psi + \phi_{Ir}(r_t - \bar{r})] \qquad (6)$$

where $\tilde{a}_{I0} = a_{I0} + a_{Ir}\bar{r}$. In the context of an open economy, the natural rate is simply the external natural rate of interest. [2] Again, it will be convenient to reinterpret $\tilde{a}_{I0}$ in terms of potential output. In a closed economy, investment exactly equals saving, therefore then $\tilde{a}_{I0}$ must equal $\psi Y^P = \sigma * (1-\tau)Y^P$. Finally, we may also re-write government spending as:

$$G_t = Y^P[\gamma + gp_t] \qquad (7)$$

where $a_{G0} = \gamma Y^P$ and $gp_t = GP_t / Y_t^P$.

4. Imports and Exports

Exports (supplied) and imports (demanded) are determined by their respective real prices (ratio to domestic price level):

$$RPX_t = \left[\frac{S_t \; * P_t^X}{P_t} \right] \qquad (8a)$$

$$RPIM_t = \left[\frac{S_t \; * P_t^{IM}}{P_t} \right] \qquad (8b)$$

where S_t is the nominal exchange rate (domestic currency per dollar, appreciation minus), P_t^X and P_t^{IM} are the dollar denominated world prices of exports and imports, and P_t is the domestic price level (home currency units).

---

[2] That is, real interest parity is assumed to hold in the long run.

It is straightforward to show that both of these prices are functions of the real exchange rate and the external terms of trade. To see this, recall first that the real exchange rate – the price of tradable goods relative to the domestic price level (appreciation minus) is written as:

$$Q_t = \left[\frac{S_t \; * \left(P_t^X\right)^{\nu} \left(P_t^{IM}\right)^{(1-\nu)}}{P_t} \right] \tag{9}$$

where the price $\left(P_t^X\right)^{\nu} \left(P_t^{IM}\right)^{(1-\nu)}$ of tradables is a geometric average of export and import prices. We may thus re-write real prices for exports and imports as:

$$RPX_t = \left[\frac{S_t \; * \left(P_t^X\right)^{\nu} \left(P_t^{IM}\right)^{(1-\nu)}}{P_t} \; * \frac{\left(P_t^X\right)^{(1-\nu)}}{\left(P_t^{IM}\right)^{(1-\nu)}} \right] = \left[Q_t \; * TT^{(1-\nu)} \right] \tag{10a}$$

$$RPIM_t = \left[\frac{S_t \; * \left(P_t^X\right)^{\nu} \left(P_t^{IM}\right)^{(1-\nu)}}{P_t} \; * \frac{\left(P_t^X\right)^{(-\nu)}}{\left(P_t^{IM}\right)^{(-\nu)}} \right] = \left[Q_t \; * TT^{(-\nu)} \right] \tag{10b}$$

where the external terms of trade TT is the ratio of externally determined export prices to import prices. The percentage deviation of export and import prices from some baseline value will we written as rpx_t and $rpim_t$, respectively.

Thus, equations the equations for exports and imports are:

$$X_t = Y^P[x + \eta_x * rpx_t] \tag{11a}$$

$$IM_t = Y_t^P[im + im_{cyc} * gap_t + \eta_{im} * rpim_t] \tag{11b}$$

We may use decompositions (10a) and (10b) in logarithmic form to see how relative inflation rates (the real exchange rate) and the terms of trade have impacts on the trade balance. Under a fixed exchange rate regime, the real exchange rate (again written as a deviation from its reference value) equals the inflation differential – external minus domestic:

$$q_t = \underbrace{(\pi^{EXT} - \pi_t)}_{\text{inflation differential}} \tag{12}$$

Thus, export and import functions are now rewritten:

$$X_t = Y^P[x + \underbrace{\eta_x * (\pi^{EXT} - \pi_t)}_{\text{inflation differential}} + \underbrace{\eta_x (1-\nu)\ln(TT_t)}_{\text{exogenous}}] \tag{13a}$$

$$IM_t = Y_t^P[im + \underbrace{im_{cyc} * gap_t + \eta_{im} * (\pi^{EXT} - \pi_t)}_{\text{gap and inflation differential}} - \underbrace{\eta_{im}\nu\ln(TT_t)}_{\text{exogenous}}] \tag{13b}$$

Note that the reference value for the external terms of trade is assumed to be unity. Thus, if there are no shocks to the terms of trade, $\ln(TT_t)$ will equal zero. An increase in domestic inflation relative to external inflation will discourage exports and encourage imports.

5. The output gap and inflation

Output in an open economy is defined as:

$$Y_t\Big|_{Open} = C_t + I_t + G_t + X_t - IM_t \tag{14}$$

We obtain an expression for the output gap by adding up expenditure side equations (5b), (6), (7), and (13a), minus (13b). After some manipulation and then subtracting potential output from and dividing by potential output on both sides, we to obtain:

$$\begin{aligned} gap_t\Big|_{FER} &= [(1-\sigma_{cyc})(1-\tau) - im_{cyc}] * gap_t - (1-\sigma_{cyc})tp_t + \varphi_{Ir}(r_t - \overline{r}) + gp_t \\ &+ (\eta_x - \eta_{im}) * (\pi^{EXT} - \pi_t) + [\eta_x(1-\nu) + \eta_{im}\nu] * \ln(TT_t) \end{aligned} \tag{15}$$

In a fixed exchange rate regime, the inflation rate has both an external and a domestic or "homegrown" component. The FER Phillips Curve is written:

$$\pi_t\Big|_{FER} = \pi^{EXT} + \underbrace{\frac{1}{\eta}(gap_t - ss_t)}_{\text{homegrown component}} \tag{16}$$

where η is the short-run elasticity of supply. That is, in the FER Phillips curve (PC-FER) the expected inflation rate equals the external inflation rate; for simplicity we have assumed that there is no trend appreciation or depreciation. [3]

We may substitute the PC-FER into the expression for output and solve for the fixed exchange rate IS curve (IS-FER) – a negative relationship between the output gap (on the demand side) and the *real* interest rate:

$$r_t\Big|_{IS,FER} = \bar{r} + \frac{gap_t\{1-[(1-\sigma_{cyc})(1-\tau)-im_{cyc}+\frac{(\eta_x-\eta_{im})}{\eta}]\}-gp_t+(1-\sigma_{cyc})tp_t+\frac{(\eta_x-\eta_{im})}{\eta}*ss_t-\tilde{\eta}_{nx}*\ln(TT_t)}{\varphi_{Ir}} \tag{17}$$

---

[3] To see how the Phillips Curve is derived from a supply function, consider the following expression for the quantity of goods and services supplied in the short run:

$$Y_t^S = Y^P[1+\eta(P_t - P_t^e)+ss_t]$$

Y_t^S = the quantity of output supplied at time t, P_t is the price level at time t, P_t^e is the expected price level at time t (as of t-1), $\eta \geq 0$ is the short-run elasticity of supply with respect to the price level, and ss_t is a supply shock (in percent of potential output). That is, $ss_t > 0$ may be thought of as a reduction in the marginal cost of production – a level shift. This equation is essentially Lucas' (1973) familiar supply function. If we normalize the price level in the previous period P_{t-1} to unity, we may add and subtract one from the term inside brackets to obtain:

This equation is essentially Lucas' (1973) familiar supply function. If we normalize the price level in the previous period P_{t-1} to unity, we may add and subtract one from the term inside brackets to obtain:

$$Y_t^S = Y^P[1+\eta(\pi_t - \pi_t^e)+ss_t]$$

For a small open economy with fixed exchange rates and constant real exchange rates, we may assume that the expected inflation rate equals the external rate of inflation: $\pi_t^e = \pi_t^{EXT}$. Then, after dividing both sides through by potential output and subtracting one from both sides, we note that $gap_t = Y_t^S / Y^P - 1$. By inverting that expression, we obtain the Phillips Curve in the text.

where

$$\tilde{\eta}_{nx} = \eta_x (1-\nu) + \eta_{im}\nu$$

6. Interest Rate Determination in an FER economy

Under a soft-peg FER, a country's central bank relinquishes control over its policy rate. Instead, that rate is largely determined by world interest rates plus a country-specific risk premium. We may write the nominal interest rate as:

$$i_t \,\big|_{FER,Open} = i_t^{EXT} + rp_t = r_t^{EXT} + \pi_t^{EXT} + rp_t \tag{18}$$

External financial pressures are the deviations of external interest rate and risk premium from their baseline (or natural) values:

$$efp_t = (i_t^{EXT} + rp_t) - (\overline{i}^{\,EXT} + \overline{rp}) \tag{19}$$

Thus to obtain the real rate of interest, we subtract the Phillips curve from the nominal rate, substituting in the expression for external financial pressures:

$$r_t \,\big|_{FER} = (\overline{i}^{\,EXT} + \overline{rp}) + efp_t - \pi^{EXT} - \frac{1}{\eta}(gap_t - ss_t) \tag{20}$$

Again, a country's natural rate of interest is tied to baseline external values according to:

$$\overline{r} = \overline{i}^{\,EXT} + \overline{rp} - \overline{\pi}^{EXT} \tag{21}$$

Substituting in (21) into (20) we obtain an expression for the real interest rate as a function of its natural value, plus movements in external interest rates, risk premia, and external inflation rates:

$$r_t \,\big|_{RR,FER} = \overline{r} + efp_t - [\pi^{EXT} - \overline{\pi}^{EXT}] - \frac{1}{\eta}(gap_t - ss_t) \tag{22}$$

7. Equilibrium values for core macro variables and graphical representation

As in other cases discussed in Tanner (2015), we combine IS curve (IS-FER, equation 17) with the expression for the real interest rate (22) to obtain the equilibrium output gap for an FER economy:

$$gap_t\Big|_{FER}^{eq} = \frac{\dfrac{gp_t - (1-\sigma_{cyc})tp_t + \tilde{\eta}_{nx} * \ln(TT_t)}{\varphi_{Ir}} + ss_t * \left(\dfrac{-\dfrac{(\eta_x - \eta_{im})}{\eta}}{\varphi_{Ir}} + \dfrac{1}{\eta} \right) + efp_t - [\pi^{EXT} - \overline{\pi}^{EXT}]}{den} \tag{23}$$

where the denominator term is:

$$den = \left(\frac{\{1 - [(1-\sigma_{cyc})(1-\tau) - im_{cyc} + \dfrac{(\eta_x - \eta_{im})}{\eta}]\}}{\varphi_{Ir}} + \frac{1}{\eta} \right)$$

The equilibrium real interest rate is thus:

$$r_t\Big|_{FER}^{eq} = \overline{r} + efp_t - [\pi^{EXT} - \overline{\pi}^{EXT}] - \frac{1}{\eta}(gap_t\Big|_{FER}^{eq} - ss_t) \tag{24}$$

Likewise, equilibrium inflation and real exchange rate appreciations are:

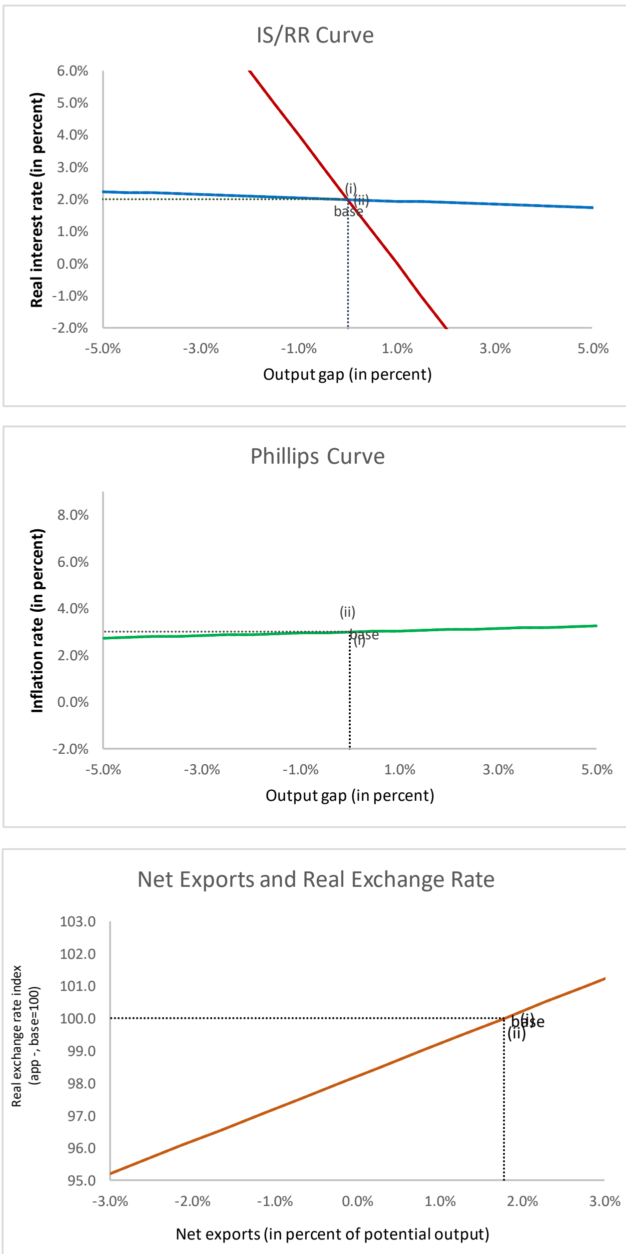
$$\pi_t\Big|_{FER}^{eq} = \pi^{EXT} + \frac{1}{\eta}(gap_t\Big|_{FER}^{eq} - ss_t) \tag{25}$$

$$q_t\Big|_{FER}^{eq} = (\pi^{EXT} - \pi_t\Big|_{FER}^{eq}) \tag{26}$$

Finally, to obtain the equilibrium trade balance, we substitute the expressions for the equilibrium real exchange rate and output gap into expressions (13a) and (13b).

Figure 1 shows a graphical depiction of four main equations that yield core macro and external sector variables for an initial equilibrium situation. The upper panel shows the IS and RR relationships in output gap (horizontal axis) and real interest rate (vertical axis) space. The red IS curve slopes downward: higher real interest rates discourage expenditures (equation 17). The blue real interest rate or RR curve (equation 22) also slopes downward – but less steeply than the IS curve.

Figure 1
The short-run model: a graphical approach



The IS curve – downward sloping red line – reflects equilibrium in the market for goods and services.

The RR curve shows the real interest rate: for a constant nominal external interest rate, the inflation rate increases with the output gap. Thus, RR slopes *downward*.

The Phillips Curve reflects a constant external inflation rate plus a homegrown component which rises with the output gap (graph assumes no supply shock).

The Net Export line shows an improvement in net exports when the real exchange rate depreciates – an increase in the real exchange rate index – for a given value of the output gap.

It is easy to see why the slope is downward: while the nominal interest rate is assumed to be exogenously given (taken from external markets) the inflation rate must rise with the output gap (according to Phillips relationship (16). Equilibrium output and real interest rate are show at the intersection of the IS and RR lines.

The middle panel shows the green Phillips Curve relationship in output gap (horizontal axis) and inflation rate (vertical axis) space. The gentle upward slope of the Phillips Curve relationship reflects a homegrown component to inflation that is assumed to be small. The equilibrium inflation rate corresponds to the equilibrium output gap – as shown above.

The bottom panel shows the orange net export relationship in net export (horizontal axis) and real exchange rate (vertical axis, appreciation minus) space. The upward slope of net export line reminds us that exports are encouraged and imports are discouraged when the real exchange rate depreciates – an upward movement along the vertical axis.

Note also that the equilibrium gap equation (23) can be broken down into contributions to the output gap. These are conveniently written as these five components, as shown in Figure 1. We may review these component-by-component: (i) An increase in the *external rate of inflation* will reduce the real interest rate, thus stimulating demand and output; (ii) a favorable *supply shock*, by reducing domestic prices, will increase the relative prices of exports and imports – a depreciation of the real exchange rate. This channel implies that a favorable shock to domestic supply will cause the IS curve to shift to the right: an increase in demand and output. However, there is also a paradoxical effect that works in the other direction: a decrease in domestic inflation will increase the real interest rate – an upward shift in the RR curve – that will reduce expenditure.

Regarding (iii) *fiscal shocks*, an increase in government spending or a decrease in taxes will cause the IS curve to shift to the right: an increase in demand and output; (iv) adverse *external financial pressures* – an increase in the external interest rate or the risk premium – will cause the RR curve to shift upwards, thus reducing domestic expenditures and output; (v) a favorable movement in the *terms of trade* will increase the relative price of exports. Such a shock will be reflected in a rightward shift of the IS curve – more demand and output, and an improvement in net exports.

To further illustrate how the model works, consider two alternative scenarios. Under alt (i), the government increases spending with the objective of expanding output, but with no adverse reactions from external financial markets; under alt (ii), assuming the same fiscal stimulus, external financial markets respond adversely – the external risk premium rises.

Figure 2 show the model dashboard (a spreadsheet based version of this model is available online at ______). In the upper portion of the dashboard, shocks to government expenditure which equal 1% of potential output are shown for alt (i) and alt (ii). That fiscal expansion is also shown graphically in the upper panel of Figure 3 – it is a rightward shift of the red IS curve.

Under alt (ii) only, a 120 basis point (1.2%) increase in the risk premium is also shown (Figure 2, upper portion). That external financial tightening is shown in Figure 3, upper panel as a leftward shift of the blue RR curve, from the solid blue to the dotted blue line.

Figure 2

External Shocks and the Output Gap: A Contributions Approach

External inflation component$= \dfrac{-[\pi^{EXT} - \bar{\pi}^{EXT}]}{den}$

An increase in the external rate of inflation will reduce the real interest rate, thus stimulating demand and output.

$$\text{Supply shock component} = \frac{ss_t * \left(\dfrac{-\dfrac{(\eta_x - \eta_{im})}{\eta}}{\varphi_{Ir}} + \dfrac{1}{\eta} \right)}{den}$$

A favorable supply shock, by reducing domestic prices, will increase the relative prices of exports and imports – a depreciation of the real exchange rate. This channel implies that a favorable shock to domestic supply will cause the IS curve to shift to the right: an increase in demand and output. However, working in the other direction, a decrease in domestic inflation will increase the real interest rate – an upward shift in the RR curve – that will reduce expenditure.

$$\text{Fiscal policy component} = \frac{\dfrac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}}}{den}$$

An increase in government spending or a decrease in taxes will cause the IS curve to shift to the right: an increase in demand and output.

$$\text{External financial pressures componnet} = \frac{efp_t}{den}$$

External financial tightening – an increase in the external interest rate or the risk premium – will cause the RR curve to shift upwards, thus reducing domestic expenditures and output.

$$\text{External terms of trade component} = \frac{\dfrac{\tilde{\eta}_{nx} * \ln(TT_t)}{\varphi_{Ir}}}{den}$$

A favorable movement in the terms of trade will increase the relative price of exports. Such a shock will be reflected in a rightward shift of the IS curve – more demand and output, and an improvement in net exports.

To further illustrate how the model works, consider two alternative scenarios. Under alt (i), the government increases spending with the objective of expanding output, but with no adverse reactions from external financial markets; under alt (ii), assuming the same fiscal stimulus, external financial markets respond adversely – the external risk premium rises.

Figure 3 shows the model dashboard (a spreadsheet based version of this model is available online at http://www.evanctanner.com/simple-models). In the upper portion of the dashboard, shocks to government expenditure which equal 1% of potential output are shown for alt (i) and alt (ii). That fiscal expansion is also shown graphically in the upper panel of Figure 4 – it is a rightward shift of the red IS curve.

Under alt (ii) only, a 120 basis point (1.2%) increase in the risk premium is also shown (Figure 2, upper portion). That external financial tightening is shown in Figure 3, upper panel as a leftward shift of the blue RR curve, from the solid blue to the dotted blue line.

The impacts of the fiscal and external shocks on output and the real interest rate that, as told by the IS/RR diagram, are straightforward: the increase in GDP brought about by a fiscal stimulus (shown by point (i) in the graph) is nearly wiped away by the external financial tightening (as reflected in point (ii)).

Likewise, the impacts fiscal and external shocks on the inflation rate, as told by Phillips Curve in the middle panel of Figure 4, are also straightforward. Likewise, under alt (i), inflation rises – albeit only minimally so since the homegrown component of inflation is assumed to be small (See Appendix for parameter values; the high value of η ensures that the homegrown effect on inflation will be small.).

Finally, the impacts of fiscal and external shocks on net exports, are shown in the bottom panel of Figure 4. Under alt (i), as the output gap rises, the orange line shifts left – from the solid line to the dotted line. This along with a small real appreciation are reflected in a worsening of the trade balance. Under alt(ii), as the higher external interest rates squeeze out expenditures, the orange line again now shifts to the right – from the dotted line to the dashed line. This, along less appreciated exchange rate (compared to alt (i)), means that the deterioration of net exports under alt (i) is nearly reversed under alt (ii).

Numerical estimates of the impacts of fiscal stimulus and adverse external financial pressures are shown for the output gap in the lower panel of Figure 3 and for other variables (core macro and expenditure components) in Figure 5. In the lower portion of the Figure 3, the fiscal policy component shows that the fiscal expansion – both alt (i) and alt (ii) -- added to the output gap by 0.86%. Since there is considerable leakage of demand into imports – as the calculation of net exports reveals below (lower portion of Figure 4) – the fiscal multiplier is less than unity. In fact, the leakage into imports is dominates any increase in demand brought about by lower real interest rates (see calculation in upper portion of Figure 4). Next, note that under alt (ii), the adverse financial shock reduces output by -.62%; hence, after the crowding out that was due to external financial tightening, the multiplier fell from its initial value to 0.24. Note also that while net exports deteriorate under alt (i) from 1.8% of GDP to 1.4% of GDP, the forced adjustment under alt (ii) nearly returns net exports to their baseline value (1.8%).

Figure 3
The short-run model: dashboard and output gap

In percent of potential output		Shocks -- Expenditure		
		base	alt(i)	alt(ii)
Gov't Spending	gp_t	0.0%	1.0%	1.0%
Tax Measures (one-off)	tp_t	0.0%	0.0%	0.0%

In percent		Shocks - Supply / Expected inflation		
Supply shock (% of Y^P)	ss_t	0.0%	0.0%	0.0%
External Inflation (gap w.r.t. baseline)	$[\pi^{EXT}-\bar{\pi}^{EXT}]$	0.0%	0.0%	0.0%

In percent		Shocks - External financial pressures (efp)		
Total external financial pressures		0.0%	0.0%	1.2%
Real interest rate -- dev.from baseline	$r_t^{EXT}-\bar{r}^{EXT}$	0.0%	0.0%	0.0%
Risk premium -- dev.from baseline	$rp_t-\overline{rp}$	0.0%	0.0%	1.2%

In percent	Shocks - External Terms of Trade (ln(TT))		
TT log deviation from baseline	0.0%	0.0%	0.0%
Export price log dev	0.0%	0.0%	0.0%
Import price log dev	0.0%	0.0%	0.0%

Calculation of equlibrium output gap -- component by component

(a) External inflation component

$$\text{External inflation component}=\frac{-[\pi^{EXT}-\bar{\pi}^{EXT}]}{den}$$

(a)	0.0%	0.0%	0.0%

(b) Supply shock component

$$\text{Supply shock component}=\frac{ss_t*\left(\frac{-\frac{(\eta_x-\eta_{im})}{\eta}}{\varphi_{Ir}}+\frac{1}{\eta}\right)}{den}$$

(b)	0.0%	0.0%	0.0%

(c) Fiscal component

$$\text{Fiscal policy component}=\frac{\frac{gp_t-(1-\sigma_{cyc})tp_t}{\varphi_{Ir}}}{den}$$

(c)	0.0%	0.9%	0.9%

(d) External financial pressure component

$$\text{External financial pressures componnet}=\frac{efp_t}{den}$$

(d)	0.0%	0.0%	-0.6%

(e) External terms of trade component

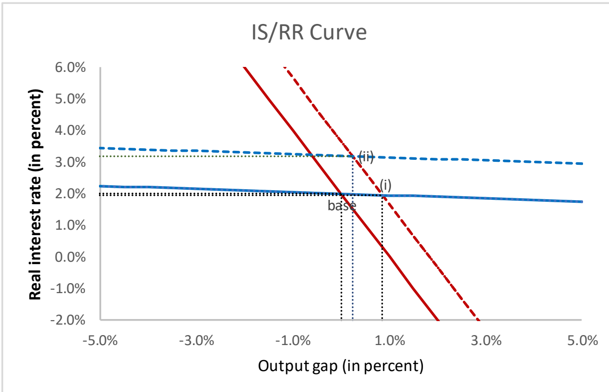
$$\text{External terms of trade component}=\frac{\frac{\tilde{\eta}_{nx}*\ln(TT_t)}{\varphi_{Ir}}}{den}$$

(e)	0.00%	0.00%	0.00%

Output gap (a)-(b)+(c)+(d)+(e)	0.0%	0.9%	0.2%

$$gap_t\Big|_{FER}^{eq}=\frac{\frac{gp_t-(1-\sigma_{cyc})tp_t+\tilde{\eta}_{nx}*\ln(TT_t)}{\varphi_{Ir}}+ss_t*\left(\frac{-\frac{(\eta_x-\eta_{im})}{\eta}}{\varphi_{Ir}}+\frac{1}{\eta}\right)+efp_t-[\pi^{EXT}-\bar{\pi}^{EXT}]}{den}$$

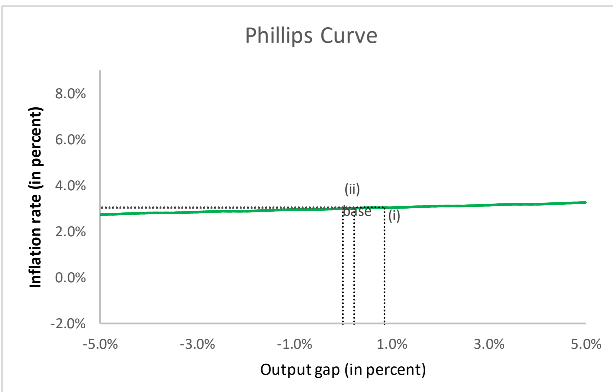
Figure 4
The short-run model: graphical illustration of scenarios



Alt (i) and alt (ii): fiscal stimulus reflected in rightward shift of red IS curve.

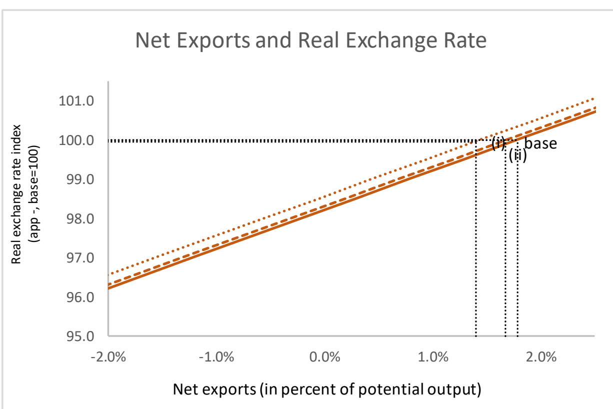
Alt (i): no external financial tightening, interest rate declines, substantial increase in output.

Alt (ii): external financial tightening reflected in leftward shift in blue RR curve from solid to dashed; increase in interest rates reduces expenditures and nearly eliminates the increase in output under alt (i).



Alt (i): increase in homegrown inflation which corresponds to additional demand.

Alt (ii): As increase in interest rates reduces expenditures and nearly eliminates the increase of inflation under alt (i).



Alt (i): fiscal expansion causes output gap to increase – a leftward shift in orange net export line from solid to dotted; this, along with a small appreciation of the real exchange rate both contribute to trade balance deterioration.

Alt (ii): As increase in interest rates reduces output gap – a rightward shift of orange net export line from dotted to dashed which nearly eliminate the deterioration of net exports under alt(i).

Figure 5
The short-run model: core macro variables
And expenditure components

Equilibrium inflation rate

$$\pi_t\big|^{eq}_{FER} = \pi^{EXT} + \frac{1}{\eta}(gap_t\big|^{eq}_{FER} - ss_t) \quad \underbrace{(gap_t - ss_t)}_{\text{homegrown component}}$$

Equilibrium inflation rate	3.00%	3.04%	3.01%

Equilibrium real interest rate

$$r_t\big|^{eq}_{FER} = \bar{r} + efp_t - [\pi^{EXT} - \bar{\pi}^{EXT}] - \frac{1}{\eta}(gap_t\big|^{eq}_{FER} - ss_t)$$

Equilibrium real interest rate	2.0%	2.0%	3.2%
Real price of exports (deviation from norm)	0.0%	0.0%	0.0%

$$\underbrace{rpx_t}_{\substack{\text{Relative price}\\ \text{of exports}}} = \underbrace{q_t}_{\text{Real exchange rate}} + \underbrace{(1-\nu)\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}} = \underbrace{(\pi^{EXT} - \pi_t)}_{\text{Inflation Differential}} + \underbrace{(1-\nu)\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}} =$$

Real price of imports (deviation from norm)	0.0%	0.0%	0.0%

$$\underbrace{rpim_t}_{\substack{\text{Relative price}\\ \text{of imports}}} = \underbrace{q_t}_{\text{Real exchange rate}} - \underbrace{\nu\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}} = \underbrace{(\pi^{EXT} - \pi_t)}_{\text{Inflation Differential}} \underbrace{-\nu*\ln(TT_t)}_{\substack{\text{Scaled External}\\ \text{Terms of Trade}}}$$

Real exchange rate (deviation from norm)	0.0%	0.0%	0.0%
(appreciaton -) $q_t\big|^{eq}_{FER} = (\pi^{EXT} - \pi_t\big|^{eq}_{FER})$			
Real exchange rate index	100.0	100.0	100.0
(Base = 100, app -)			
Currency-unit results			
Gross Domestic Product	18.33	18.49	18.38
Consumption	11.92	11.96	11.93
Investment	3.34	3.34	3.21
Governerment Spending	2.75	2.93	2.93
Net Exports	0.33	0.26	0.31
Exports	1.45	1.45	1.45
Imports	1.12	1.19	1.14
Net Exports/Y^P	1.8%	1.4%	1.7%
Net Exports/Y^P Baseline	1.8%	1.8%	1.8%
NX gap	0.0%	0.4%	0.1%
Demand side decomposition of output gap (percent of potential)			
Output gap	0.0%	0.9%	0.3%
Consumption	0.0%	0.2%	0.1%
Investment	0.0%	0.0%	-0.7%
Governerment Spending	0.0%	1.0%	1.0%
Net Exports	0.0%	-0.3%	-0.1%
Exports	0.0%	0.0%	0.0%
Imports	0.0%	0.3%	0.1%

8. Central bank deficit financing and international reserves

It is well-known that, by adopting a fixed exchange rate regime, the amount of financing that a central bank can provide for a government is constrained. According to the traditional monetary approach to the balance of payments, if the growth in government bond holdings relative to base money exceeds the growth rate in the demand for base money, the central bank will eventually run out of reserves. If market participants expect international reserve holdings to be depleted, they will attempt to preemptively purchase all of those reserve holdings *today* -- a speculative attack.

Even so, governments *can* rely on some central bank financing in the short term – so long as such borrowing is done in a prudent manner in the context of a sustainable long-run path for government debt. One of the goals of a traditional financial programming exercise is to gauge ex-ante the likely impacts on central bank reserve holdings for a given macroeconomic, fiscal, and financing scenario.

The model presented in this note can also be extended to illustrate such impacts. To begin, we present the implications of the government spending stimulus in alt (i) and (ii) from the standpoint of the fiscal accounts; Figure 6 shows a simplified fiscal account. [4] Note that we have added an assumption regarding government liabilities: the inherited debt from the previous period totaled 50% of output. We assume that the nominal interest rate paid on government debt is that determined in equation (18), above. Under the baseline, the primary balance is assumed to be zero while the overall balance, reflecting interest payments, is -2.5% of output. Under alt (i), the primary balance deteriorates to -.9% of output and the overall balance correspondingly falls to -3.3%. However, alt (ii) shows the impact of external financial tightening – higher risk premium and capital outflows – on the fiscal. Since the interest rate rises by 120 basis points, interest *payments* rise from 2.5% to 3.1% of output, with corresponding deteriorations in overall balances and the debt / GDP ratio.

Note that we have assumed that the financing mix under alt (ii) differs from that of either the baseline or alt (i); in the face of higher external interest rates and less abundant financing, the government will now rely on the central bank to purchase 60% of its new debt issue (under base and alt (i), the central bank does not contribute to fiscal financing).

The resources available to the government from the central bank will be determined by the market's willingness to hold liabilities of the central bank – its base money (assumed here to be mainly currency). The demand for base money is written as:

$$M_t^d = \bar{M}[1 + \pi_t + \varepsilon_{MD,gap} * gap_t + \varepsilon_{MD,i} * (i_t - \bar{i}^{EXT}) + aut_{MD,t}] \qquad (27)$$

[4] Note that these numbers are expressed in nominal terms that are obtained by multiplying real terms by the price index (baseline price level times one plus inflation rate).

Figure 6
The short-run model: fiscal variables

Fiscal Accounts (Nominal terms)		base	alt(i)	alt(ii)
Revenues		2.75	2.77	2.76
Primary Expenditures		2.75	2.93	2.94
Primary Balance		0.00	-0.16	-0.18
Interest Payments		0.46	0.46	0.57
Overall Balance		-0.46	-0.62	-0.75
Debt		9.63	9.79	9.91
Memo: Government debt from previous period	9.167			
In percent of GDP	50.0%			
Memo: Nominal Interest Rate		5.0%	5.0%	6.2%
In percent of GDP				
Revenues		15.0%	15.0%	15.0%
Primary Expenditures		15.0%	15.9%	16.0%
Primary Balance		0.0%	-0.9%	-1.0%
Interest Payments		2.5%	2.5%	3.1%
Overall Balance		-2.5%	-3.3%	-4.1%
Debt		52.5%	52.9%	53.9%
Financing (Nominal terms)		0.46	0.62	0.75
Domestic		0.00	0.00	0.45
Of Which Central Bank		0.00	0.00	0.45
External		0.46	0.62	0.30
Memo: Domestic/Total		0	0	0.6
Memo: Central Bank/Domestic		0	0	1
In percent of GDP				
Financing		2.5%	3.3%	4.1%
Domestic		0.0%	0.0%	2.4%
Of Which Central Bank		0.00	0.00	0.02
External		2.5%	3.3%	1.6%

where $\bar{M}$ is a reference level of base money which reflects the 'normal' velocity under the baseline, $\varepsilon_{MD,gap}$ is the income elasticity of money holdings (assumed to be unity), $\varepsilon_{MD,i} < 1$ reflects the response of money holdings to the nominal interest rate, and $aut_{MD,t}$ is an error term that captures any other autonomous shift in money holdings (assumed to be zero in these simulations).

Thus, equation (27) tells us that money demand rises one-to-one with the price level; money demand will also rise when real income rises but will fall when the interest rate rises (as an opportunity cost term). Therefore, this equation pins down the liabilities of the central bank; the composition of central bank assets is assumed to be exogenous and determined in part by the fiscal financing. [5]

[5] Note that, in addition to currency, other liabilities of the central bank include deposits; net worth is assumed constant for simplicity.

Figure 7

The short-run model: central bank balance sheet

Central Bank Accounts (Nominal terms)		base	alt(i)	alt(ii)
Liabilities				
Base Money		2.47	2.49	2.45
Currency		1.89	1.90	1.88
Deposits / Other		0.38	0.38	0.38
Other/Net Worth		0.20	0.20	0.20
Assets		2.47	2.49	2.45
Net International Reserves		1.24	1.26	0.69
Domestic		1.23	1.23	1.77
Government debt	1	1.00	1.00	1.45
Credit to banks		0.20	0.20	0.29
Other		0.03	0.03	0.03
RELATIVE TO BASELINE				
Change in international reserves			**0.02**	**-0.55**
in percent of GDP			**0.1%**	**-3.0%**

$$M_t^d = \bar{M}[1+\pi_t + \varepsilon_{MD,gap} * gap_t + \varepsilon_{MD,i} * (i_t - \bar{i}^{EXT}) + aut_{MD,t}]$$

Memo: Reference value of base money	$\bar{M}$	1.833
In percent of GDP		10.0%
Baseline velocity		10
Deposits etc		2.0%
Response parameters		
Money demand - output gap	$\varepsilon_{MD,gap}$	1
Money demand - inflation rate		1
Money demand - nominal intrest rate gap	$\varepsilon_{MD,i}$	-0.7

As Figure 7 shows this financing decision is reflected in a substantial increase in government debt under alt (ii) compared to the other scenarios. Next, note that net international reserves are computed as a residual: the difference between total liabilities and all other (domestic) assets. Hence, in alt (ii), net international reserves must fall.

Under alt (i), international reserves increase slightly; this reflects the additional money demand that occurred under the benign fiscal stimulus scenario. However, under alt (ii), where the contraction in foreign financing encouraged the government to rely more heavily on central bank financing, reserves fall dramatically – by just over 3% of output.

9. Summary, Conclusions, and Possible Extensions

This note presented a simple model of a small open economy with a fixed exchange rate. The model illustrated the impacts of both domestic and external shocks on core macroeconomic variables. Results were displayed both in a graphical format (similar to many textbooks) and as quantitative estimates for core macroeconomic variables and expenditure components, including the trade balance. The model was also extended to illustrate linkages government financing and reserve flows.

The model is meant to be used both as a *didactic* and a *practical* tool for policy makers in small economies that have adopted a soft-peg. To this end, a small spreadsheet based model is available as a companion to the paper. The model may be extended to include cyclical adjustments to the fiscal balance, and complete treatments of the fiscal, monetary and balance of payments (external) sectors.

REFERENCES

Blanchard, Olivier, 2009. "The State of Macro," *Annual Review of Economics*, *Annual Reviews*, vol. 1(1), pages 209-228, 05.

Krugman, Paul, 2000, "How Complicated Does the Model Have To Be?" *Oxford Review of Economic Policy* (Vol 16, No. 4), pp. 33 -42

Lucas, Robert, 1973, "Some International Evidence on Output-Inflation Tradeoffs,"*American Economic Review*, vol. 63, issue 3, pages 326-34

Rose, Andrew K., 2011, "Exchange Rate Regimes in the Modern Era: Fixed, Floating, and Flaky," UC Berkeley Haas School of Business Working Paper, available online at http://faculty.haas.berkeley.edu/arose/fff.pdf

Romer, David, 2000, "Keynesian Macroeconomics without the LM Curve," *The Journal of Economic Perspectives*, Vol. 14, No. 2 (Spring), pp. 149-169

Tanner, Evan, C., 2017, "The Algebraic Galaxy of Simple Macroeconomic Models: A Hitchhiker's Guide," *IMF Working Paper;* forthcoming in *Open Economies Review*.

Walsh, Carl E. , 2002. "Teaching Inflation Targeting: An Analysis for Intermediate Macro," *The Journal of Economic Education*, vol. 33(4) (Dec.) pp 333-346

Appendix: Parameter Values for Model

Main Equations of Short-Run Model

Consumption

$$C_t = Y^P\{\underbrace{(1-\sigma)(1-\tau)}_{\text{long-run component}} + \underbrace{(1-\sigma_{cyc})[(1-\tau)*gap_t - TP_t]}_{\text{short-run component}}\}$$

$(1-\tilde{\sigma}) = (1-\sigma)(1-\tau)$	0.65	$(1-\sigma_{cyc})$	0.30	$(1-\sigma_{cyc})(1-\tau)$	0.26

Investment

$$I_t = Y^P[\psi + \varphi_{Ir}(r_t - \overline{r})]$$

ψ	0.18	φ_{Ir}	-0.6

Government

$$G_t = Y^P[\tau + gp_t]$$

τ	0.15

Exports of goods and services

$$X_t = Y^P[x + \underbrace{\eta_x * (\pi^{EXT} - \pi_t)}_{\text{inflation differential}} + \underbrace{\eta_x (1-\nu)\ln(TT_t)}_{\text{exogenous}}]$$

η_x	0.50	ν	0.03

Imports of goods and services

$$IM_t = Y_t^P[im + \underbrace{im_{cyc} * gap_t + \eta_{im} * (\pi^{EXT} - \pi_t)}_{\text{gap and inflation differential}} - \underbrace{\eta_{im}\nu\ln(TT_t)}_{\text{exogenous}}]$$

im_{cyc}	0.40	η_{im}	-0.50

IS Curve (open economy)

$$r_t\Big|_{IS,FER} = \overline{r} + \frac{gap_t\{1-[(1-\sigma_{cyc})(1-\tau) - im_{cyc} - \frac{(\eta_x - \eta_{im})}{\eta}]\} - gp_t + (1-\sigma_{cyc})tp_t + \frac{(\eta_x - \eta_{im})}{\eta} * ss_t - \tilde{\eta}_{nx} * \ln(TT_t)}{\varphi_{Ir}}$$

$\tilde{\sigma}_{cyc} = 1-(1-\sigma_{cyc})(1-\tau)$	0.75	$\eta_{nx} = \eta_x - \eta_{im}$	1.00
$\tilde{\eta}_{nx} = \eta_x(1-\nu) + \eta_{im}\nu$	0.47	$\{1-[(1-\sigma_{cyc})(1-\tau) - im_{cyc} - \frac{(\eta_x - \eta_{im})}{\eta}]\}$	0.30
$\frac{(\eta_x - \eta_{im})}{\eta}$	0.05		

Phillips Curve (Inverse Short Run Supply Function, fixed exchange rate)

$$\pi_t \big|_{FER} = \pi^{EXT} + \frac{1}{\eta} \underbrace{(gap_t - ss_t)}_{\text{homegrown component}}$$

η	20.00	θ	0.00

Real Interest Rate (RR) schedule, fixed exchange rate)

$$r_t \big|_{FER} = \bar{r} + efp_t - [\pi^{EXT} - \bar{\pi}^{EXT}] - \frac{1}{\eta}(gap_t - ss_t)$$

$b_{RR\pi} = \beta_\pi$	0.00	$b_{RRgap} = \frac{(\beta_\pi - 1)}{\eta} + \beta_{gap}$	-0.05	$b_{RRss} = \frac{(\beta_\pi - 1)}{\eta}$	-0.05
$b_{RRefp} = [b_\pi - 1]\theta + 1$		1.00			

Equilibrium output gap (fixed exchange rate)

$$gap_t \Big|_{FER}^{eq} = \frac{\dfrac{gp_t - (1-\sigma_{cyc})tp_t + \tilde{\eta}_{nx} * \ln(TT_t)}{\varphi_{Ir}} + ss_t * N1 + efp_t - [\pi^{EXT} - \bar{\pi}^{EXT}]}{den}$$

Denominator

$$den = \left(\frac{\{1 - [(1-\sigma_{cyc})(1-\tau) - im_{cyc} - \dfrac{(\eta_x - \eta_{im})}{\eta}]\}}{\varphi_{Ir}} + \frac{1}{\eta} \right)$$

-1.94